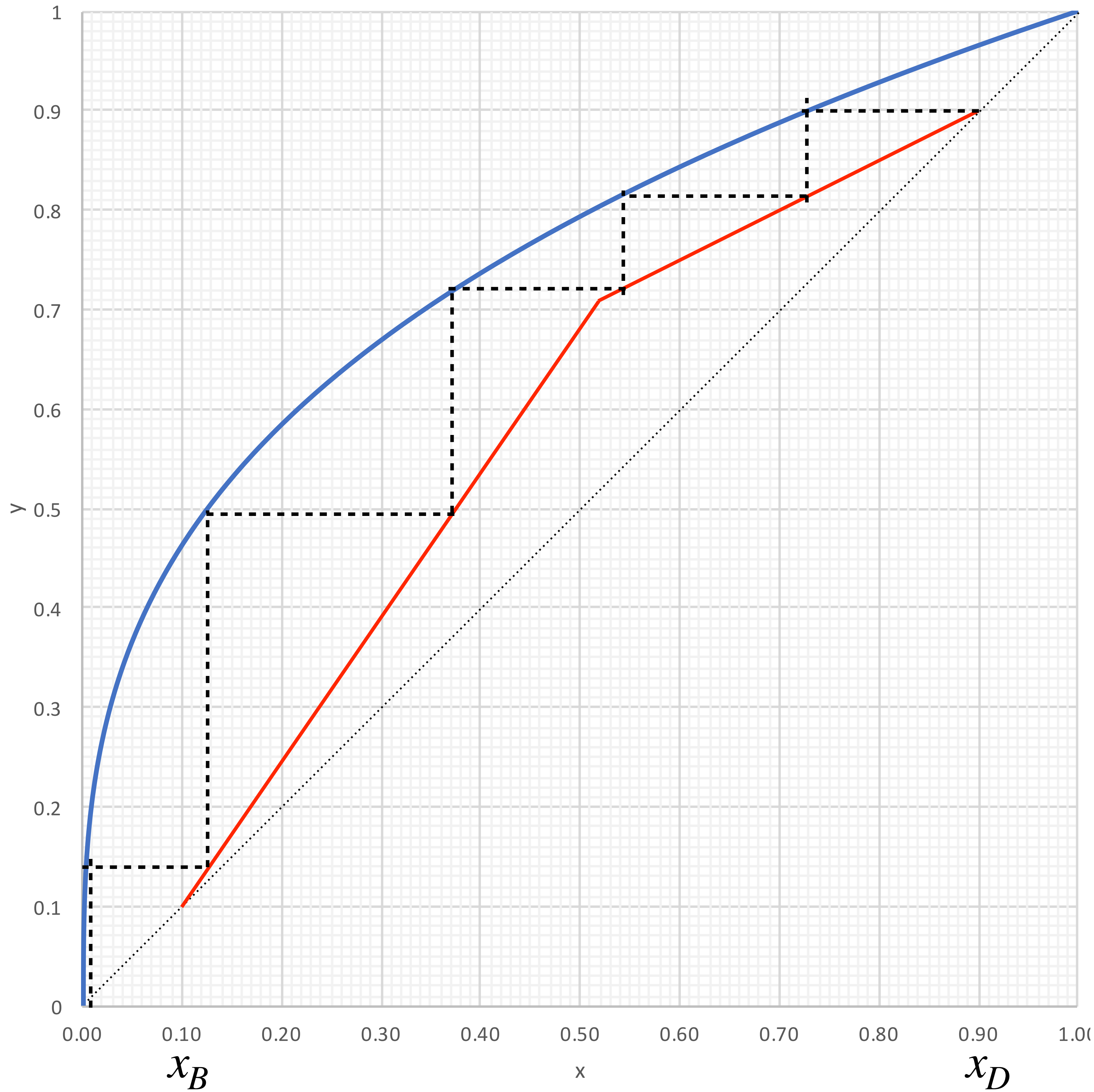
Lecture 4

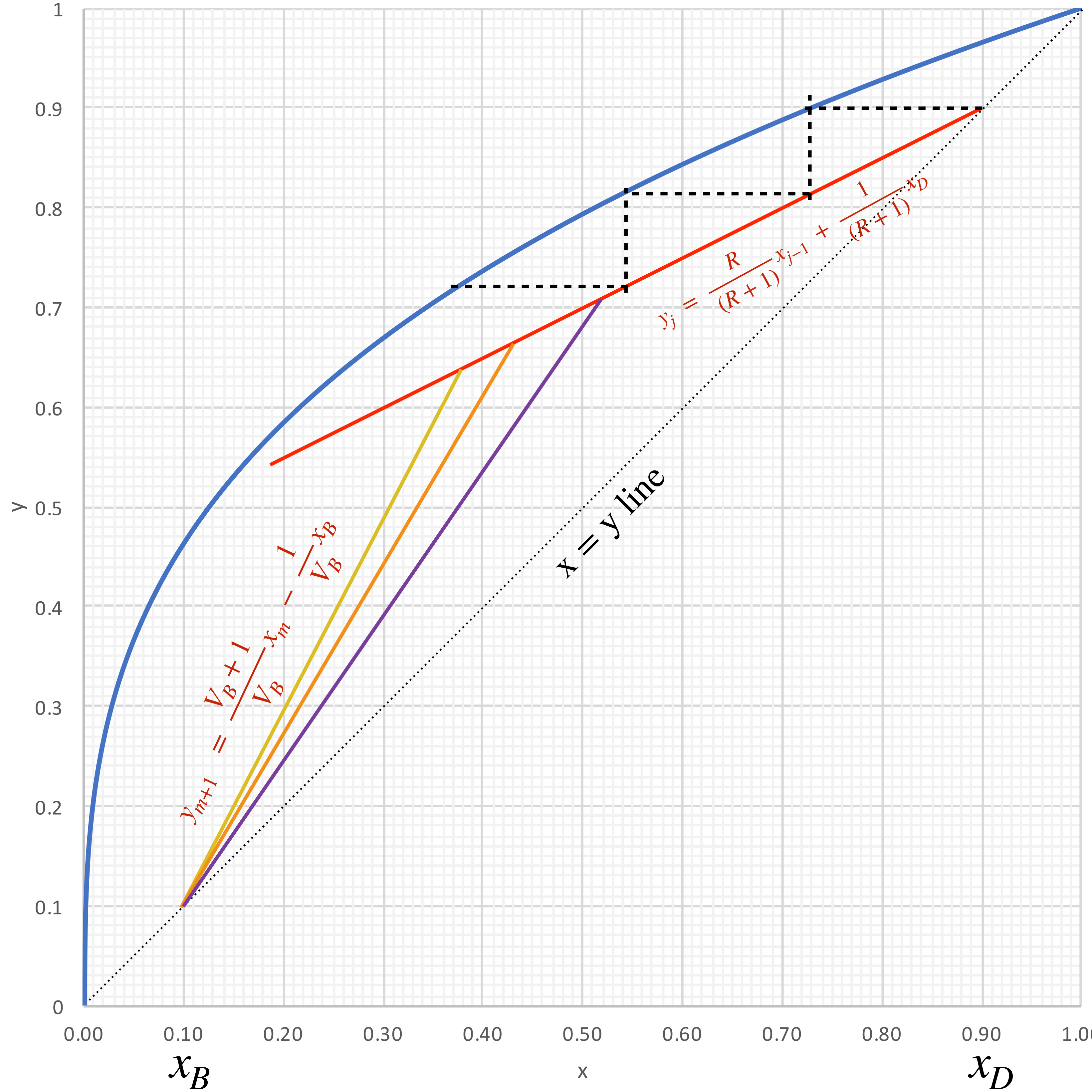
Intended Learning Outcome:

1. To understand and calculate the minimum reflux ratio.
2. To calculate minimum number of stages for distillation.
3. To analyze relationship between reflux ratio and number of stages. To understand optimal reflux ratio.
4. To analyze the case with partial condenser and total reboiler.
5. Analyze the case of multiple feed.

Where will you introduce the feed



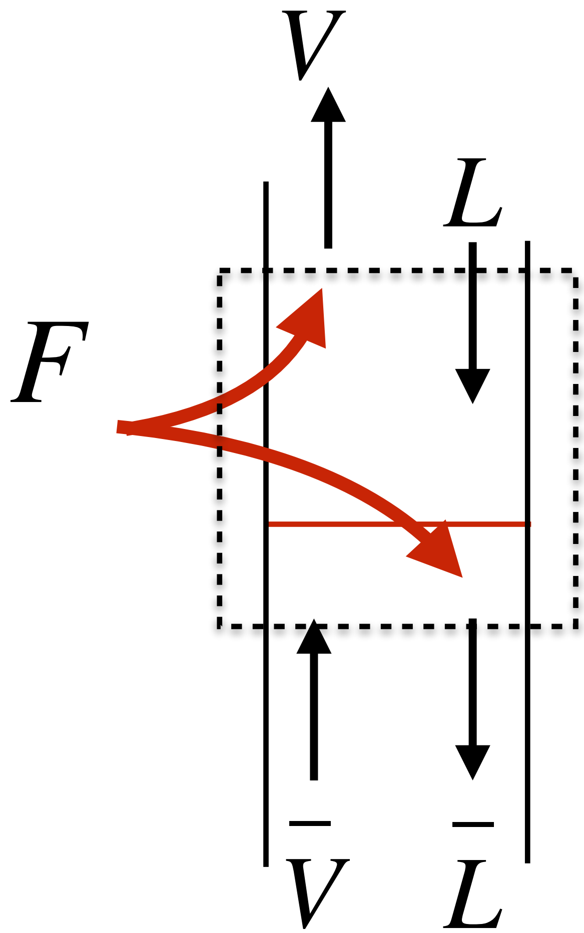
Often, V_B is not provided and you only have R (reflux ratio)



In this case, typically feed quality is specified

Feed condition

Feed quality, q



Simple observations

$$V \geq \bar{V}$$

$$\bar{L} \geq L$$

Feed quality, q, is defined by

$$q = \frac{\bar{L} - L}{F}$$

Feed condition	**q**
Subcooled liquid	
Saturated liquid	
Two-phase mixture	
Saturated vapor	
Superheated vapor	

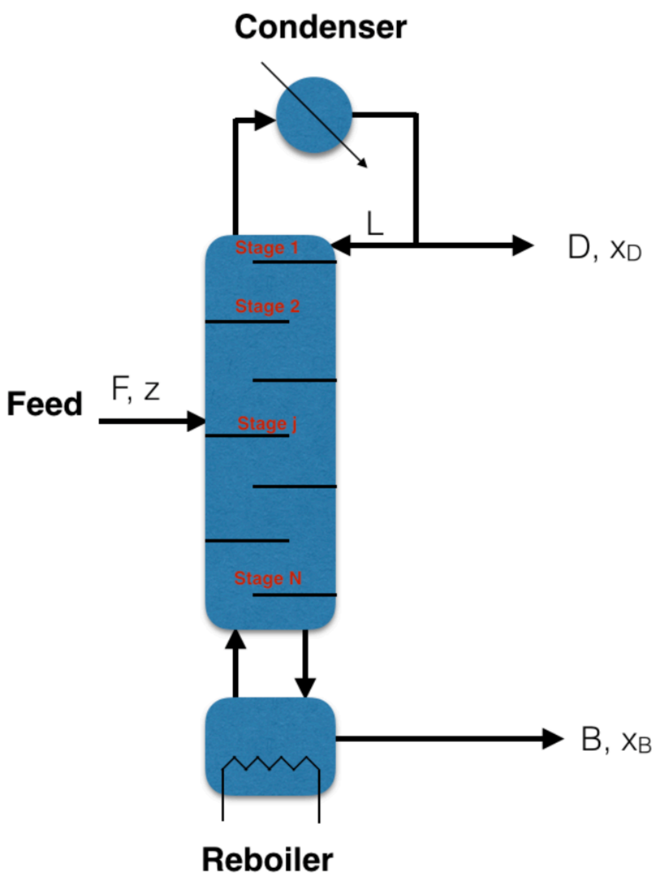
Overall balance

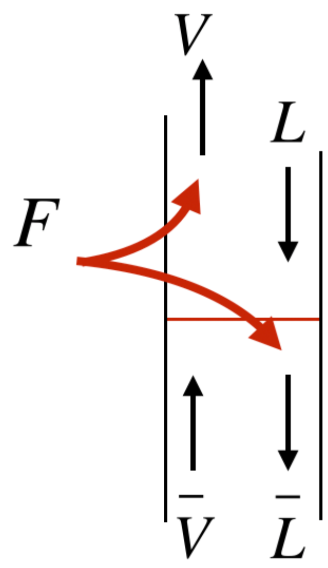
$$F + L + \bar{V} = V + \bar{L}$$

$$\Rightarrow q = \frac{\bar{L} - L}{F} = \frac{F - (V - \bar{V})}{F}$$

$$\Rightarrow q = \frac{\bar{L} - L}{F} = 1 - \frac{V - \bar{V}}{F}$$

The feed condition





Rectifying section $Vy = Lx + Dx_D$

Stripping section $\bar{V}y = \bar{L}x - Bx_B$

These two lines meet at the feed plate

$$\left(\bar{V}-V\right)y=\left(\bar{L}-L\right)x-Bx_B-Dx_D$$

$$\Rightarrow y = \frac{\bar{L}-L}{\bar{V}-V}x - \frac{Bx_B+Dx_D}{\bar{V}-V}$$

$$Fz = Dx_D + Bx_B$$

$$q = \frac{\bar{L}-L}{F} = 1 - \frac{V-\bar{V}}{F}$$

$$\Rightarrow \bar{L}-L = qF$$

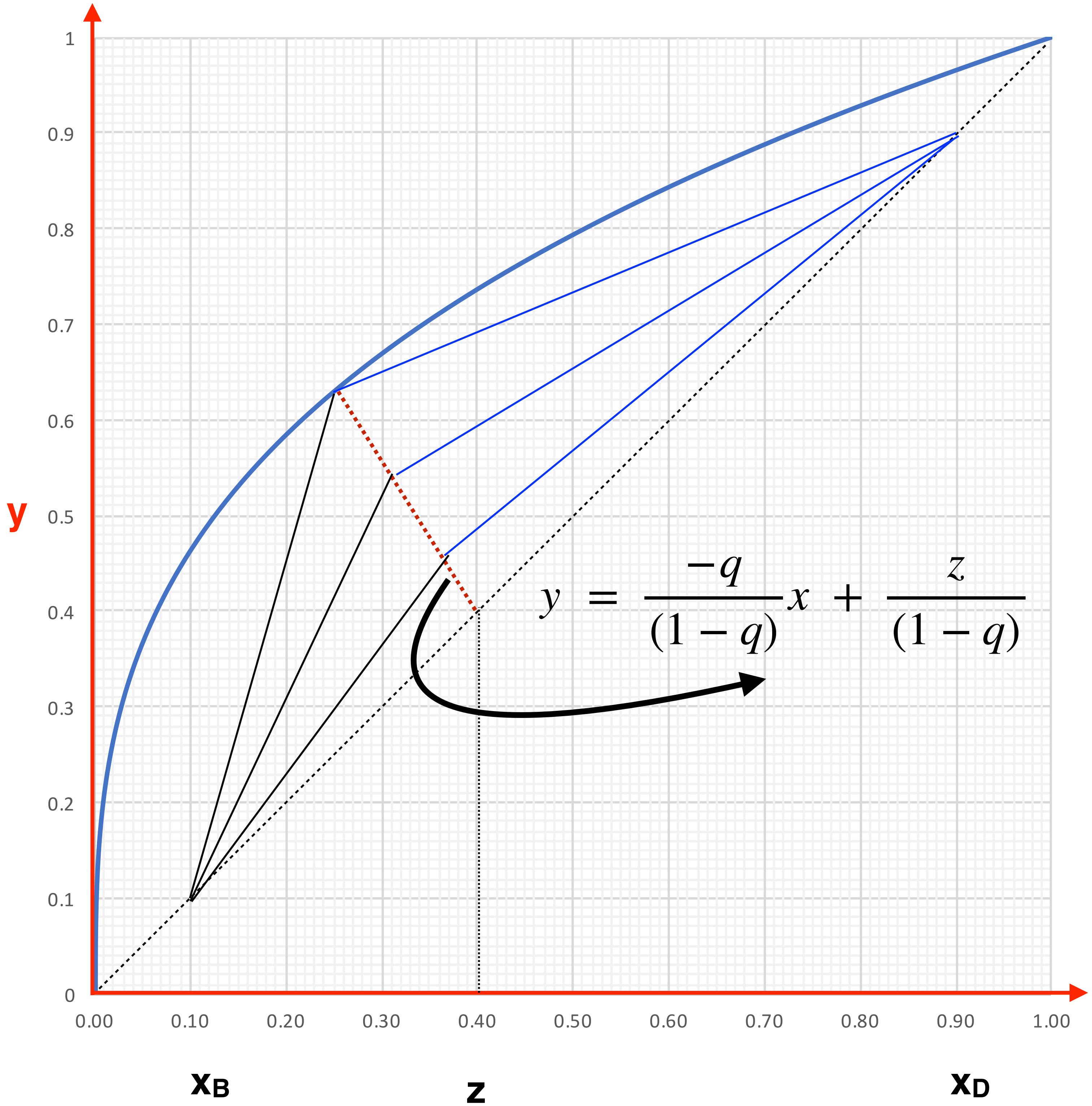
$$\Rightarrow V-\bar{V}=F-qF$$

$$\Rightarrow y = \frac{qF}{-(1-q)F}x - \frac{Fz}{-(1-q)F}$$

$$\Rightarrow y = \frac{-q}{(1-q)}x + \frac{z}{(1-q)}$$

Feed Line

The feed condition



$$y = \frac{-q}{(1-q)}x + \frac{z}{(1-q)}$$

x=y=z fits in this line

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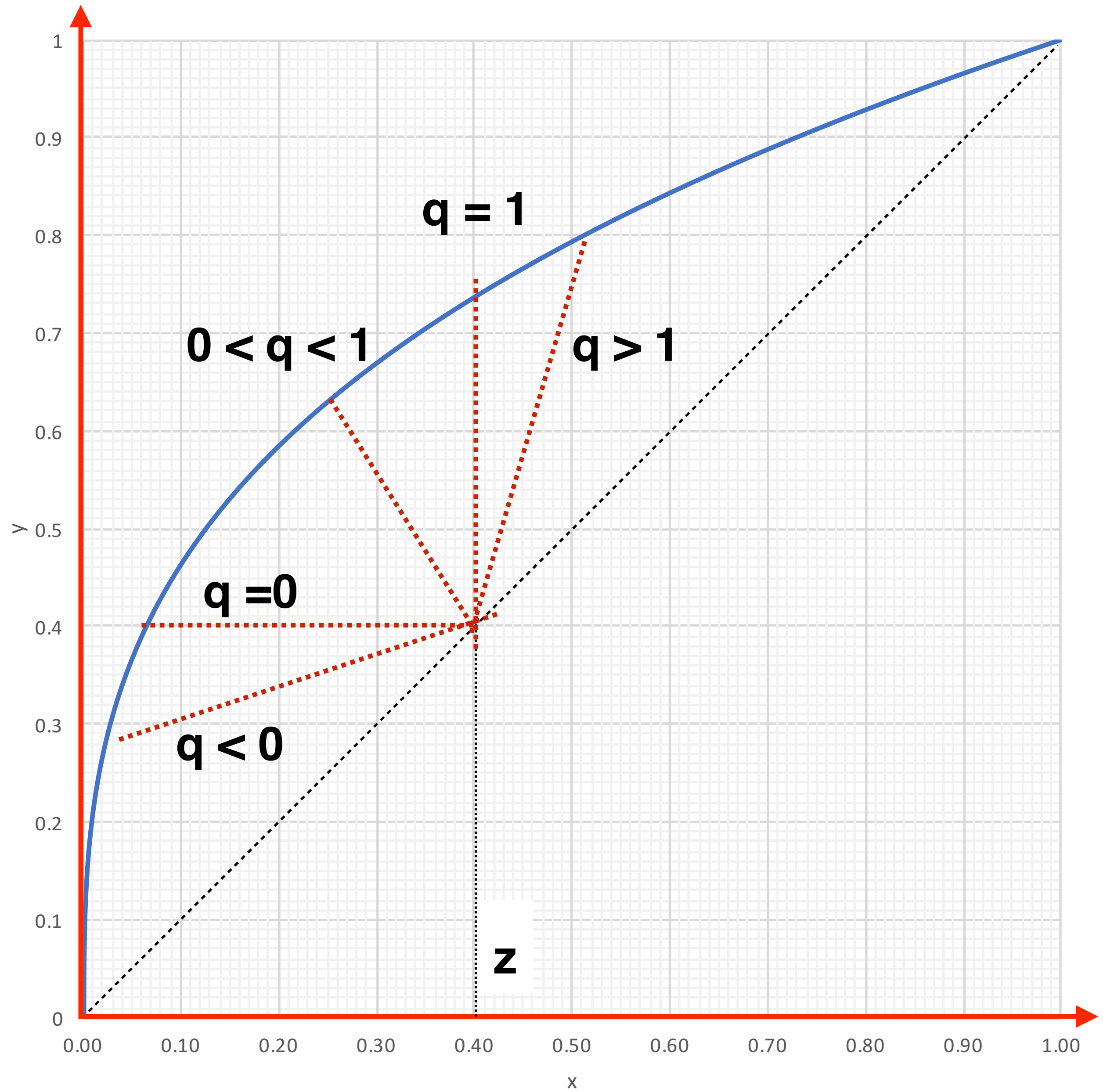
The feed condition (assign q values)

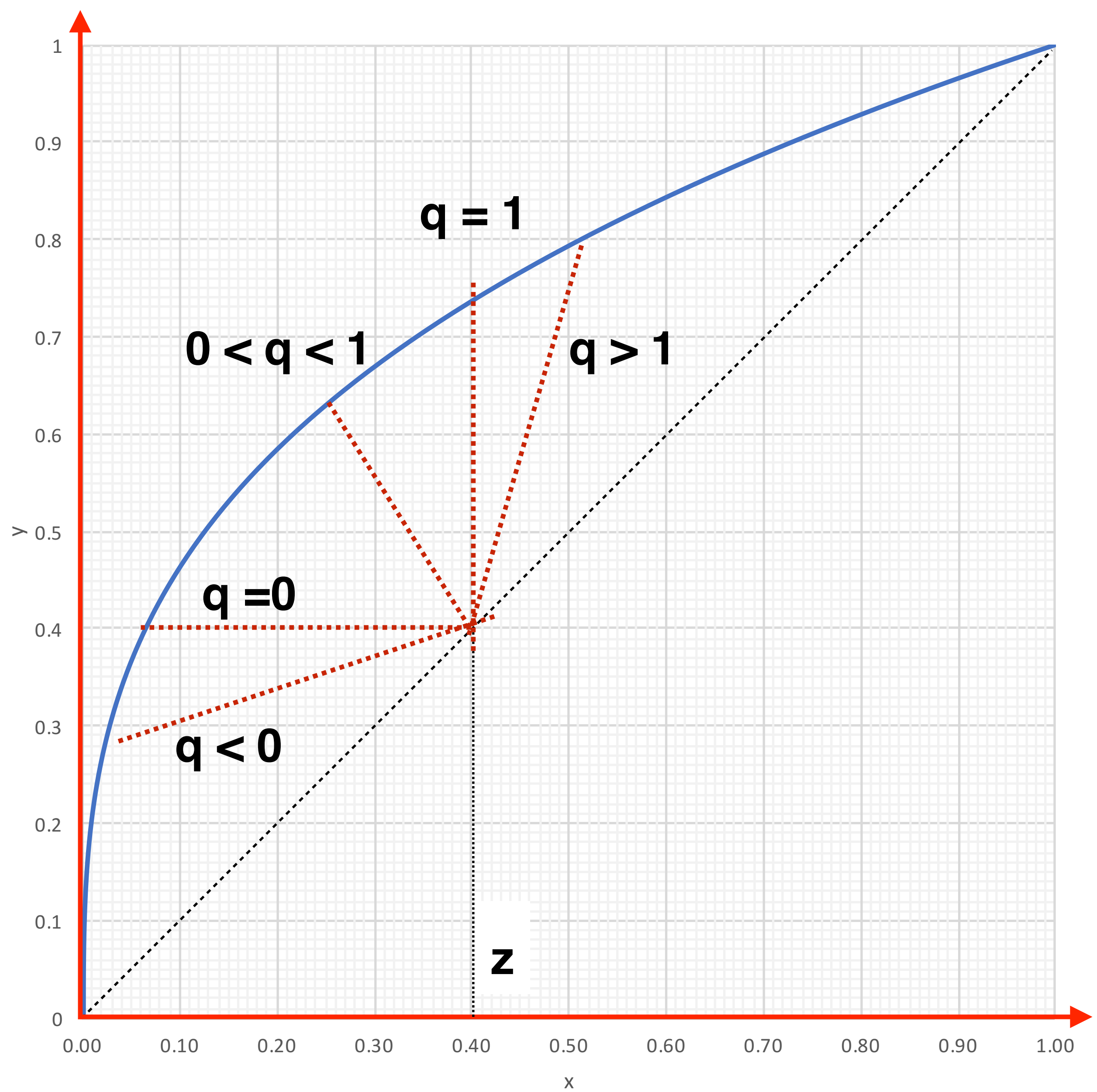


Feed condition	q
Subcooled liquid	q > 1
Saturated liquid	q = 1
Two-phase mixture	0 < q < 1
Saturated vapor	q = 0
Superheated vapor	q < 0

$$y = \frac{-q}{(1-q)}x + \frac{z}{(1-q)}$$

EPFL

The feed condition (assign q values)



Feed condition	q
Subcooled liquid	$q > 1$
Saturated liquid	$q = 1$
Two-phase mixture	$0 < q < 1$
Saturated vapor	$q = 0$
Superheated vapor	$q < 0$

$$y = \frac{-q}{(1-q)}x + \frac{z}{(1-q)}$$

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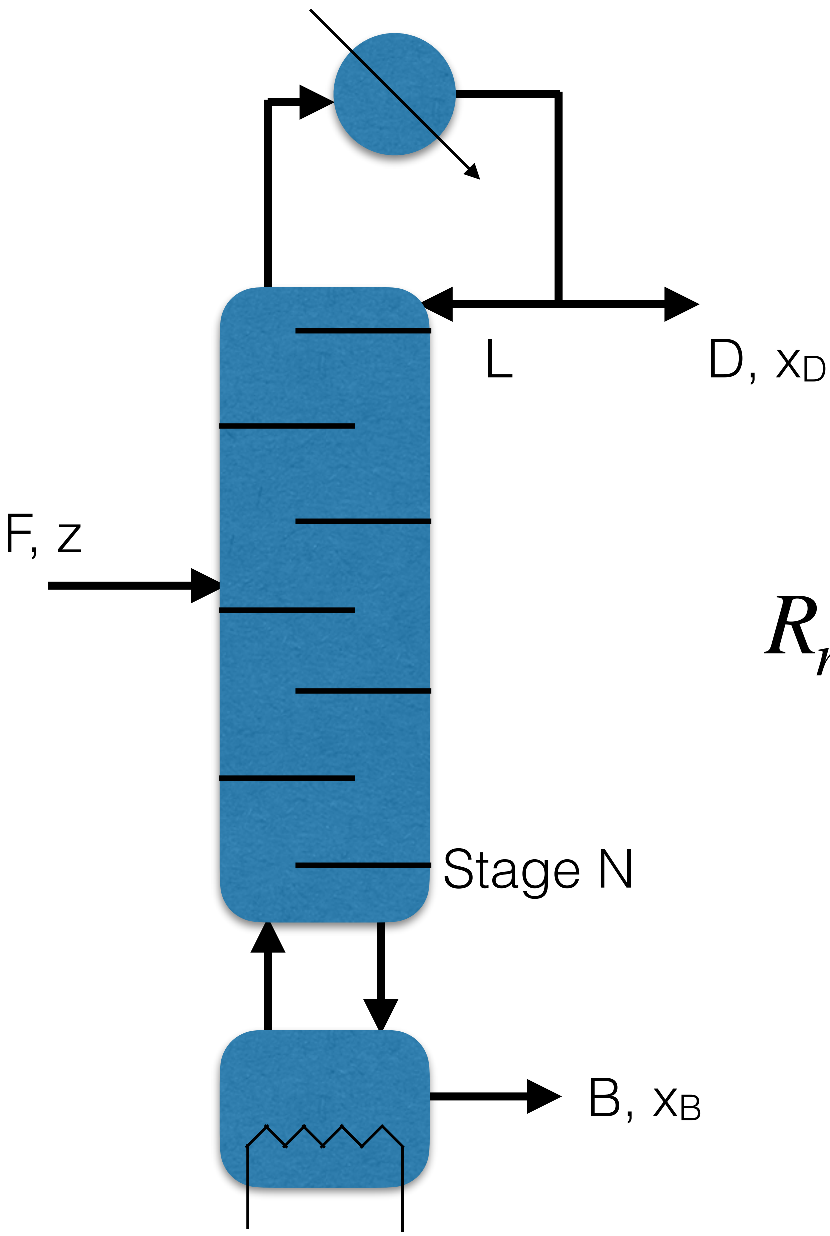
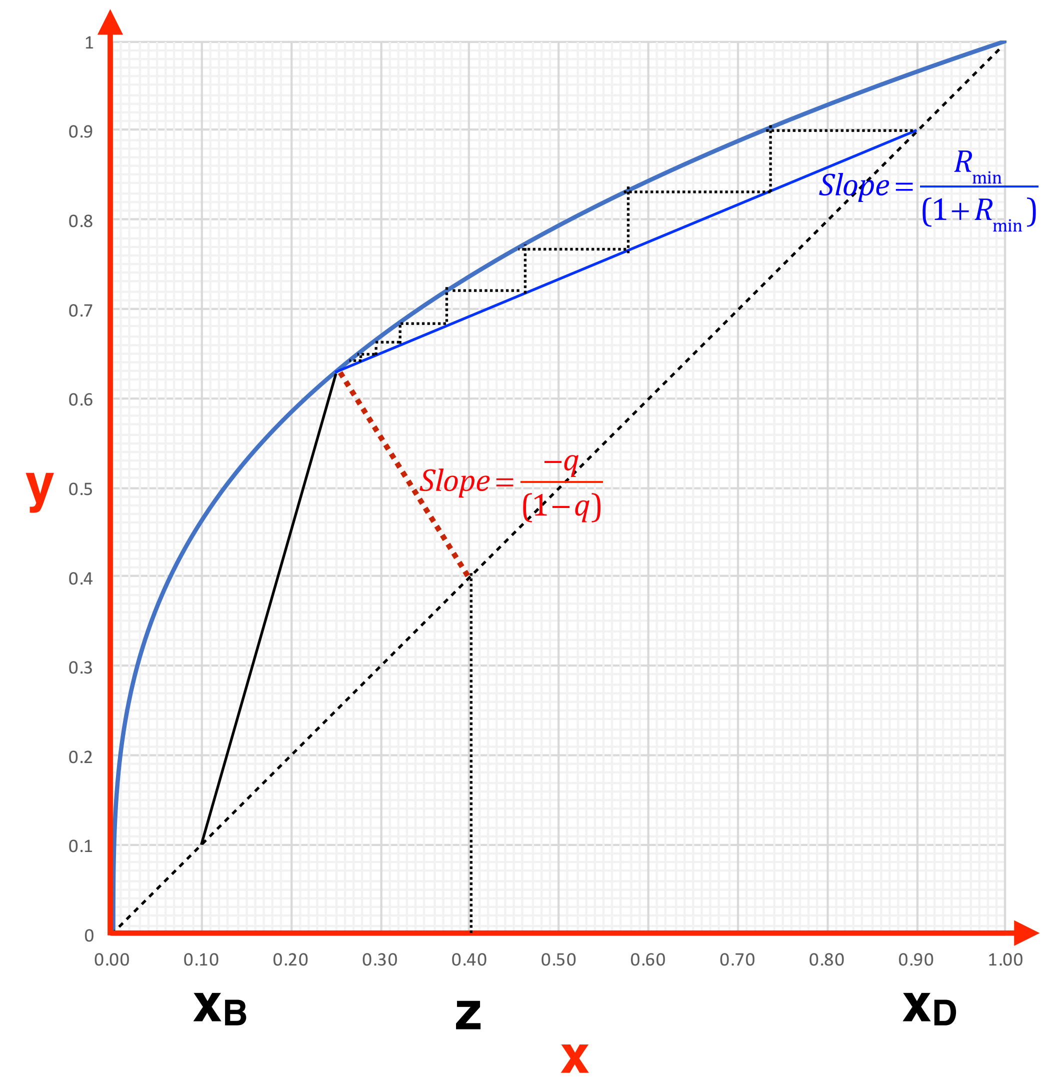
The case of minimum reflux ratio

Why would you want to decrease the reflux ratio ??



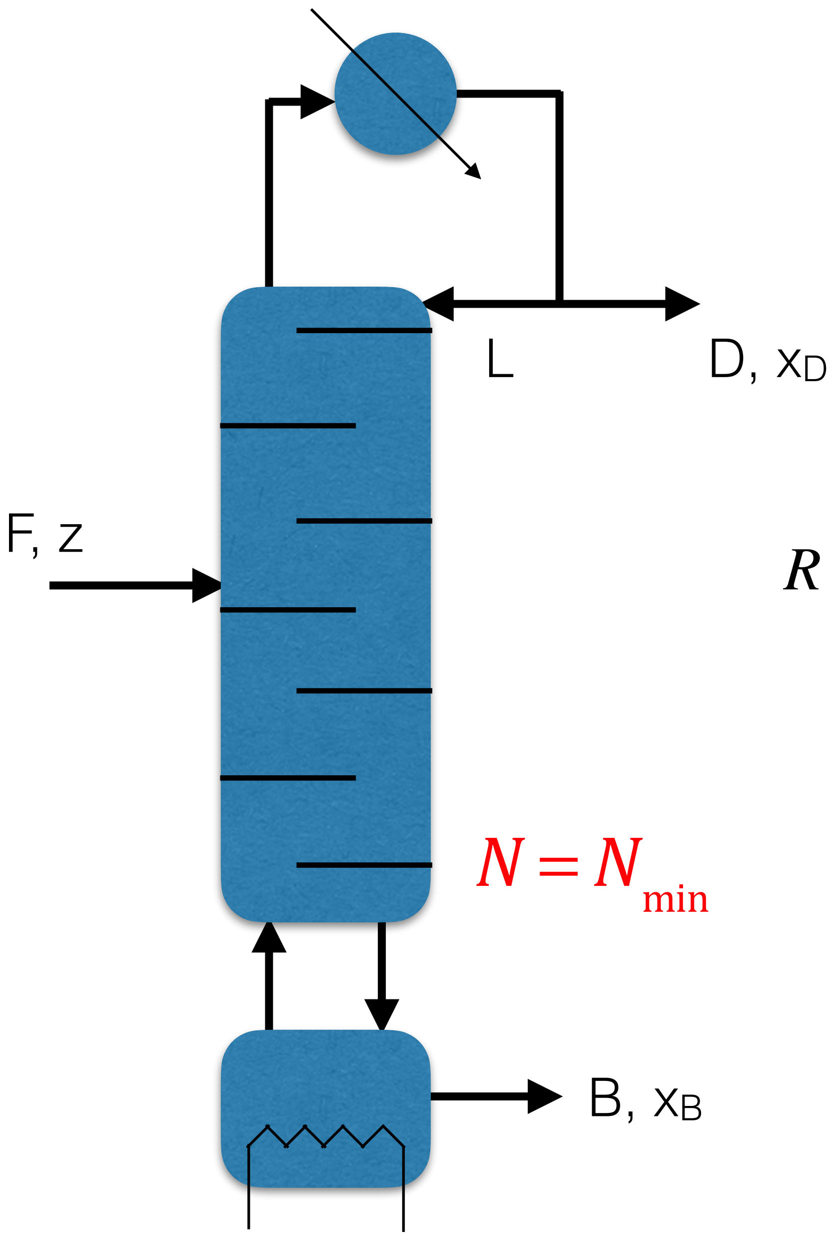
$$R_{min} = \frac{L_{min}}{D}$$



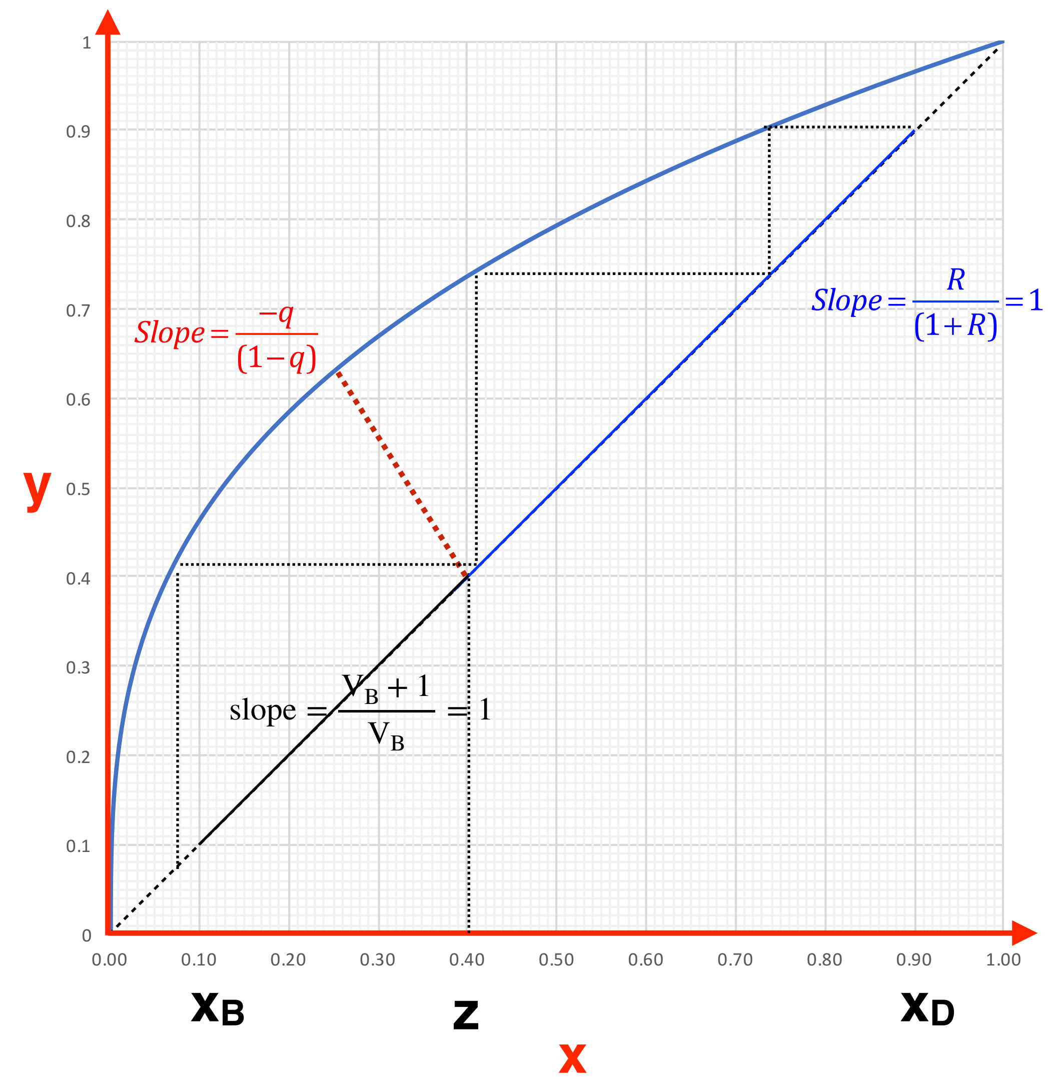
$$N = \infty$$

The case of minimum number of stages

Why would you want to decrease the number of stages?
How would we decrease number of stage?
What could be the advantages and the disadvantages of minimum stages?



$$R = \lim_{D \to 0} \frac{L}{D} \to \infty$$

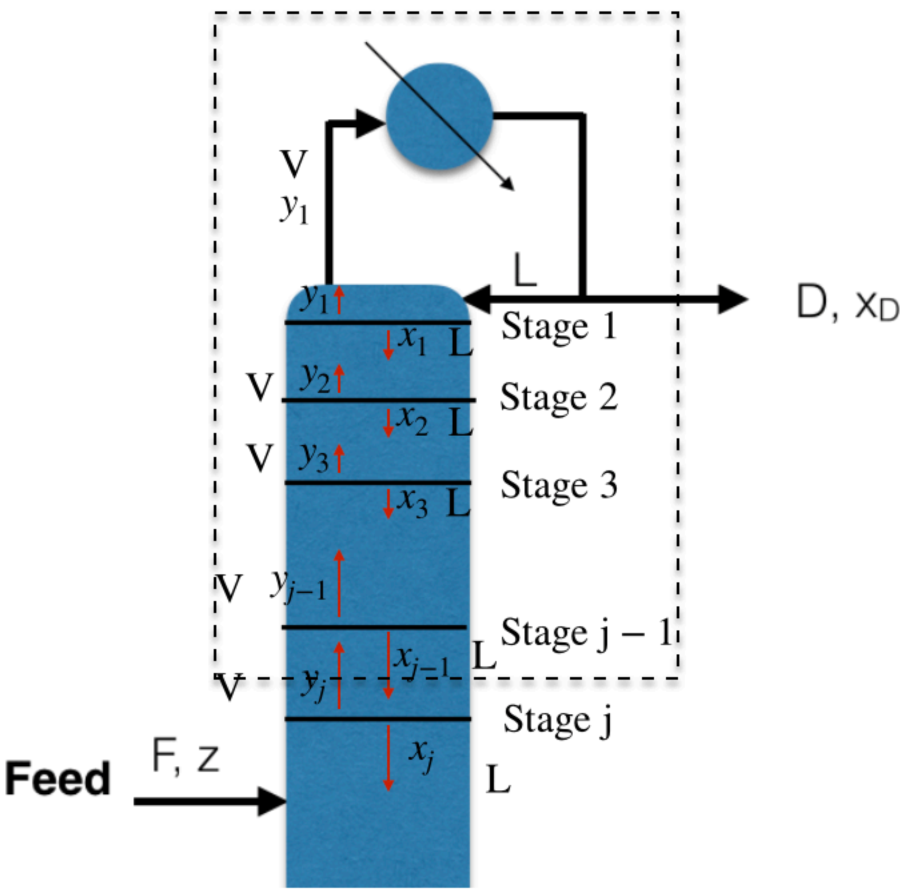


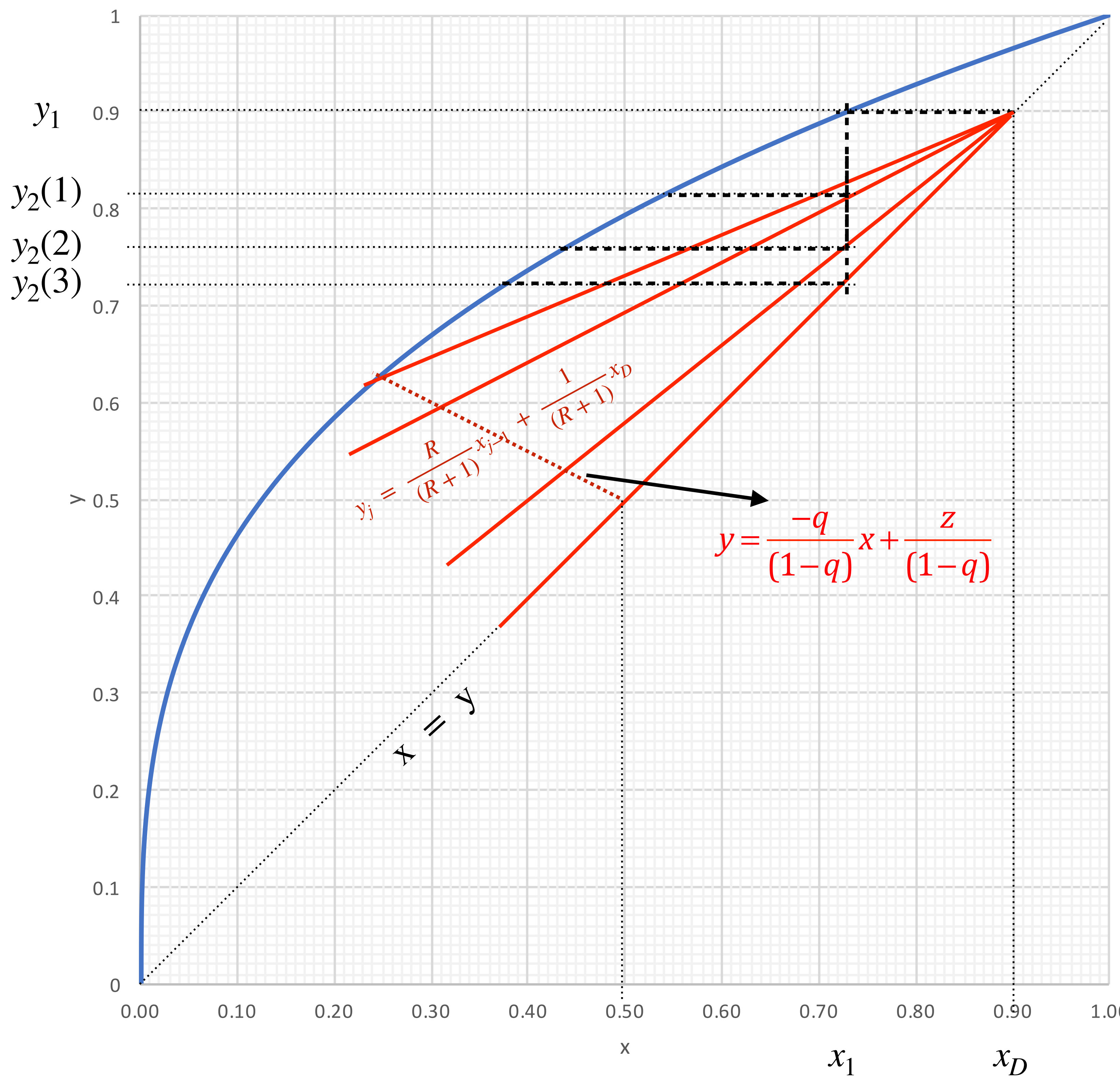
$R \to \infty$ $\quad D \to 0$

$F \to 0$

$V_B \to \infty$ $\quad B \to 0$

Understanding the effect of reflux





$$Vy = Lx + Dx_D$$

$$y = \frac{L}{V}x + \frac{D}{V}x_D$$

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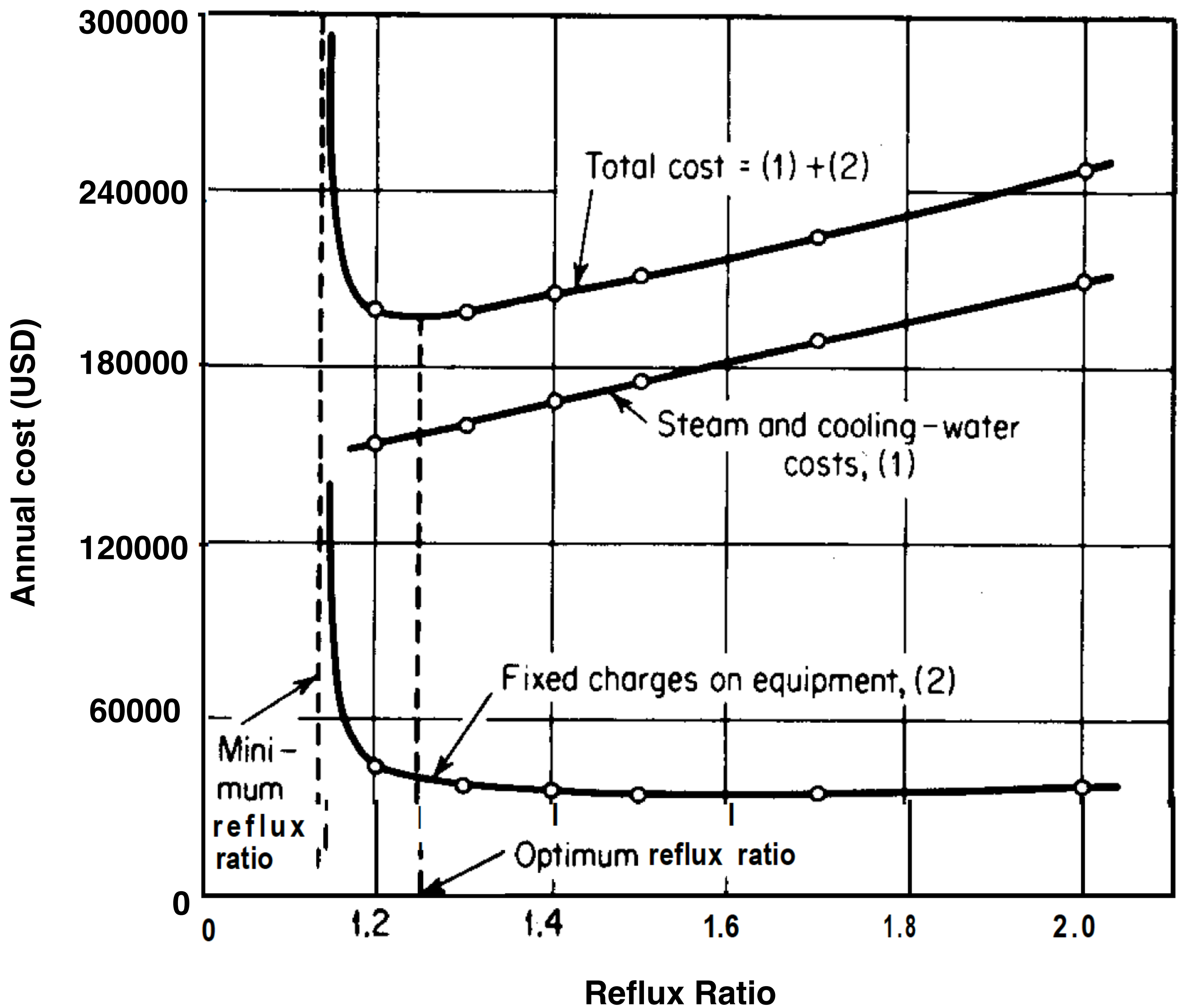
Optimization between capital cost and operating cost in distillation column



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Optimal reflux ratio

Reflux ratio	Number of stages	L	Q_C and Q_R
R_{min}	∞	Small	Small
∞	N_{min}	Large	Large

Table 7.3 Effect of Reflux Ratio on Annualized Cost of a Distillation Operation

R/R_{min}	Actual N	Diam., ft	Reboiler Duty, Btu/h	Condenser Duty, Btu/h	Annualized Cost, \$/yr Equipment	Annualized Cost, \$/yr Cooling Water	Annualized Cost, \$/yr Steam	Total Annualized Cost, \$/yr
1.00	Infinite	6.7	9,510,160	9,416,000	Infinite	17,340	132,900	Infinite
1.05	29	6.8	9,776,800	9,680,000	44,640	17,820	136,500	198,960
1.14	21	7.0	10,221,200	10,120,000	38,100	18,600	142,500	199,200
1.23	18	7.1	10,665,600	10,560,000	36,480	19,410	148,800	204,690
1.32	16	7.3	11,110,000	11,000,000	35,640	20,220	155,100	210,960
1.49	14	7.7	11,998,800	11,880,000	35,940	21,870	167,100	224,910
1.75	13	8.0	13,332,000	13,200,000	36,870	24,300	185,400	246,570

(Adapted from an example by Peters and Timmerhaus [6].)

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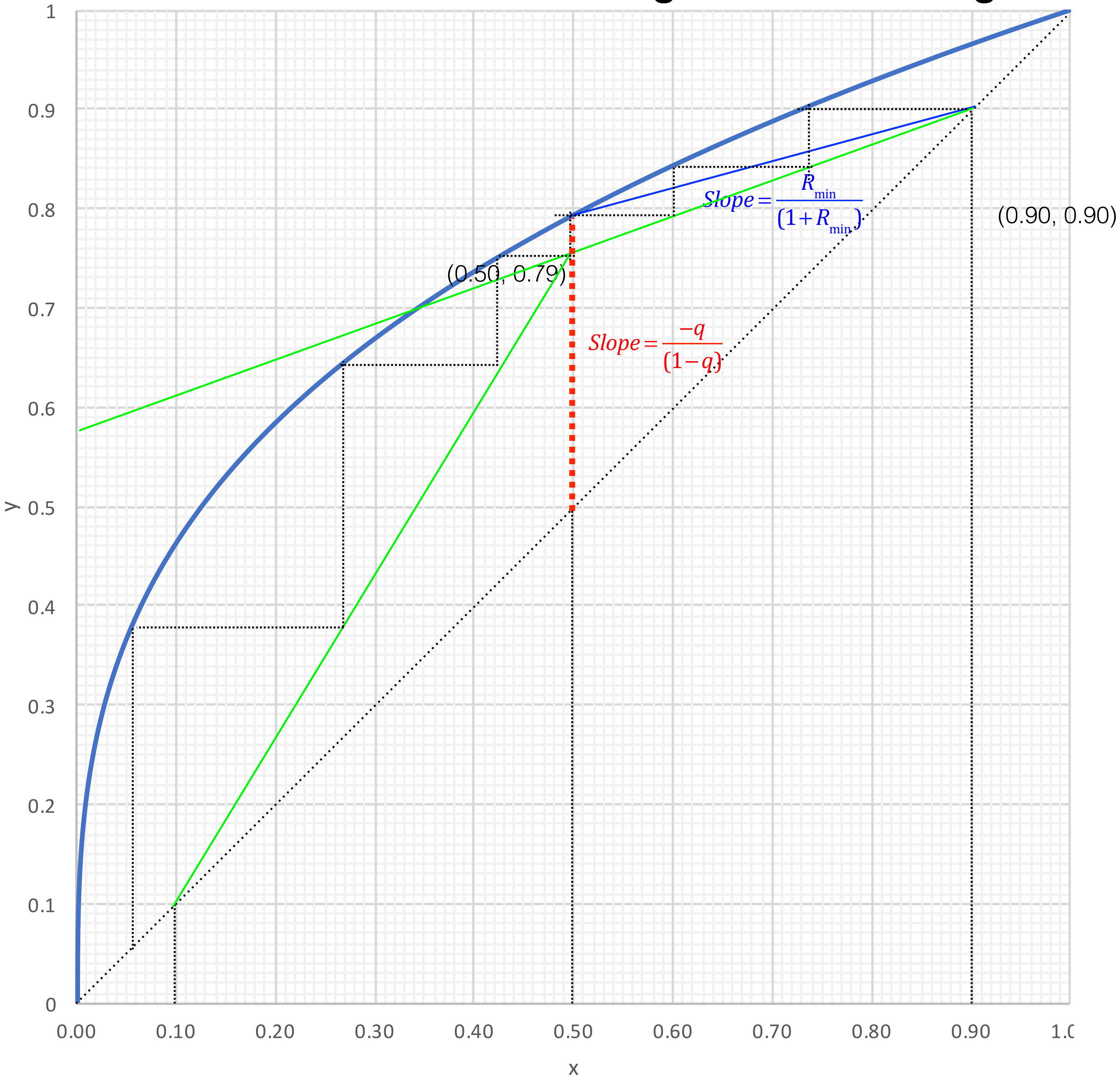
Consider the following setup. $x_D = 0.9$, $x_B = 0.1$, $z = 0.5$. Saturated liquid feed.

1. **Calculate the minimum reflux ratio.**
2. **The optimal reflux ratio was estimated to be when $R = 1.5\ R_{min}$. Calculate the number of stages.**
3. **Calculate minimum number of stages. Does it change with feed condition?**



Slope = (0.90-0.79)/(0.90-0.50)

= 0.11/0.4 =0.275

R_{min} = 0.275/(1-0.275) = 0.38

R = 1.5 R_{min} = 0.38*1.5 = 0.57

Slope = 0.57/1.57 = 0.36

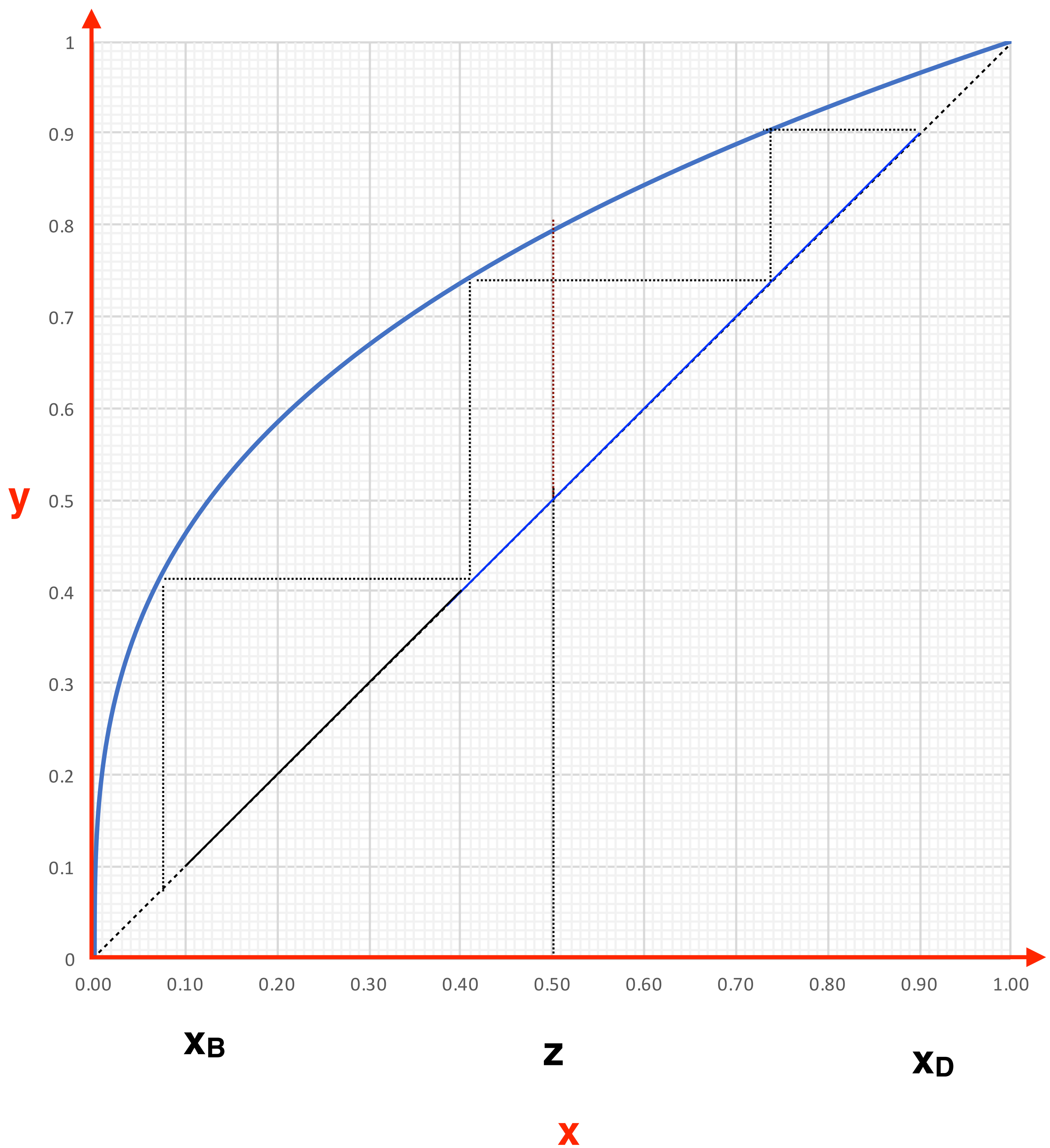
Operating line, y = 0.36 x + C

C = 0.9 - 0.36*0.9 = 0.58

Y = 0.36x +0.58

$$N = 6$$

Minimum number of stages



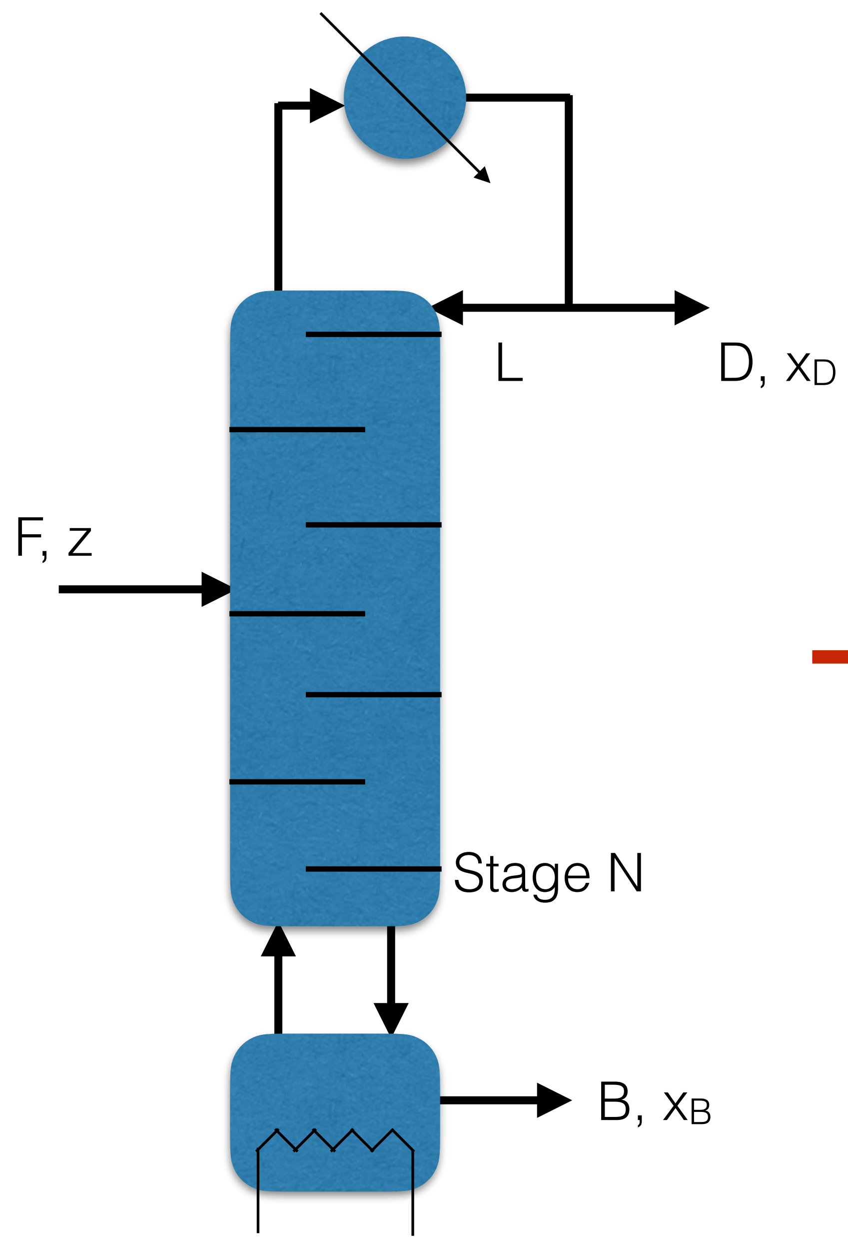
x_B z x_D

$$Slope = \frac{R}{(1+R)} = 1$$

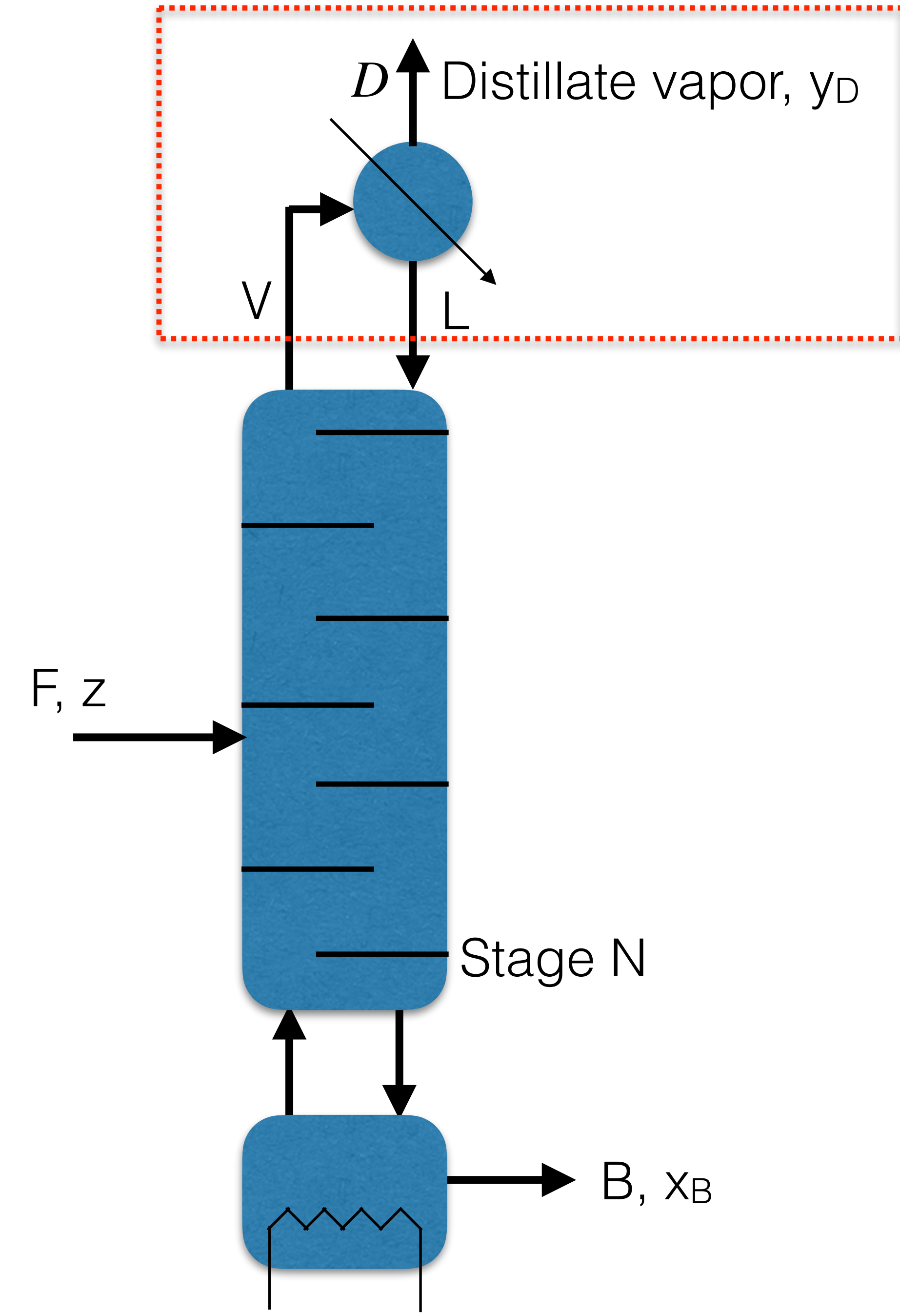
$$N = N_{\min}$$

The case of partial condenser

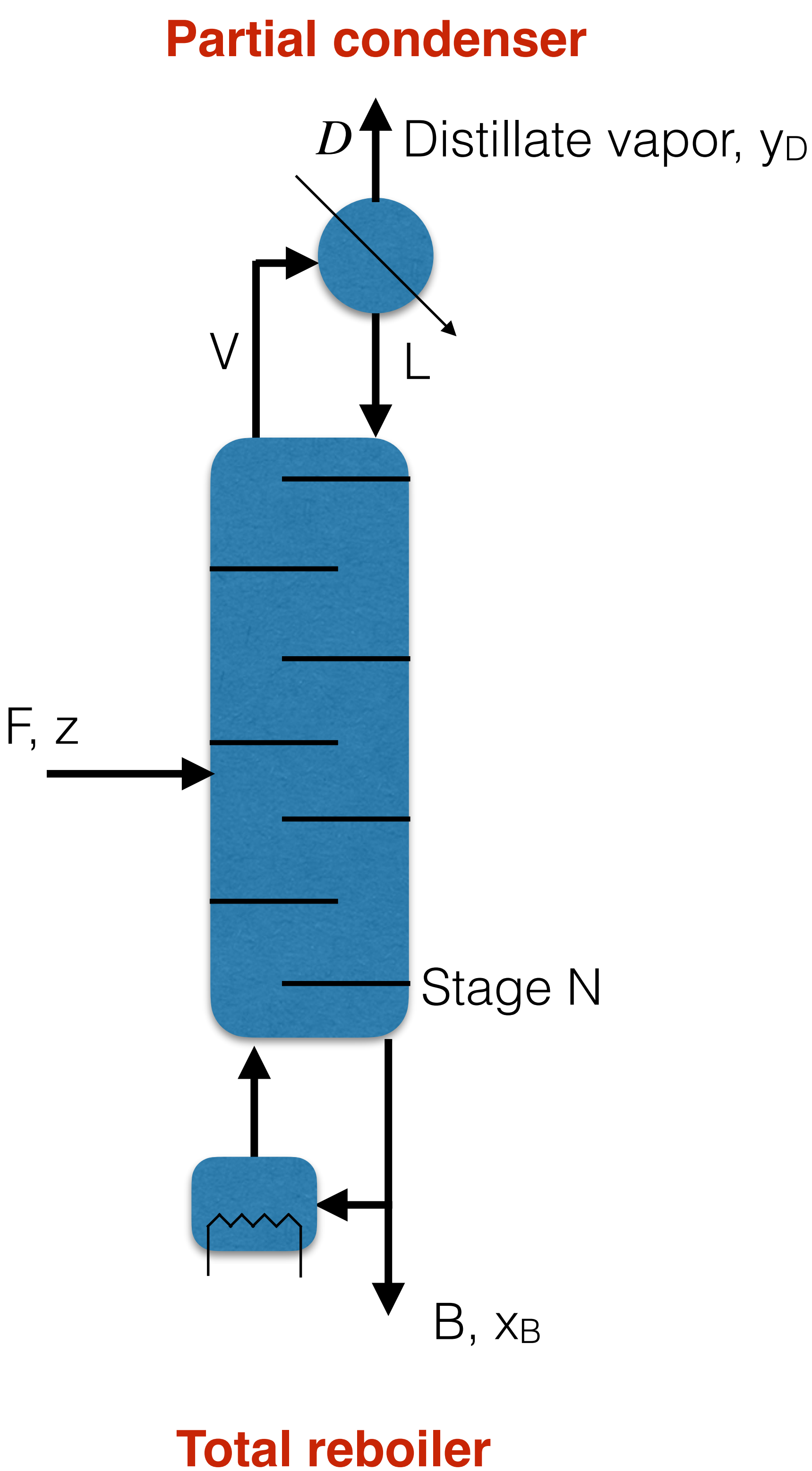
When the vapor distillate is desired



Total condenser

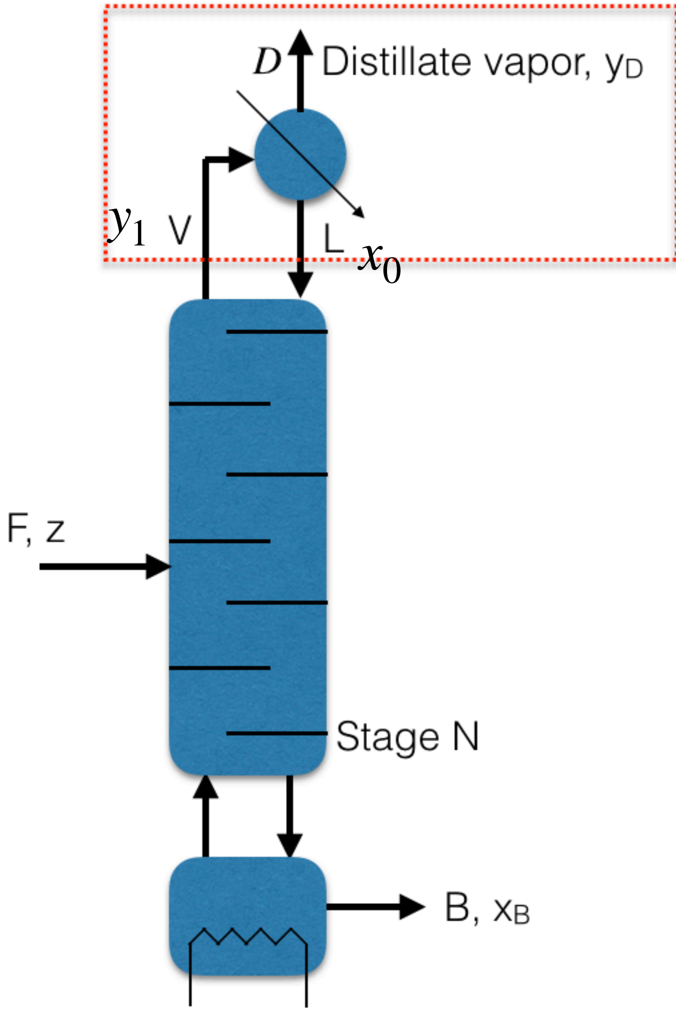


Partial condenser

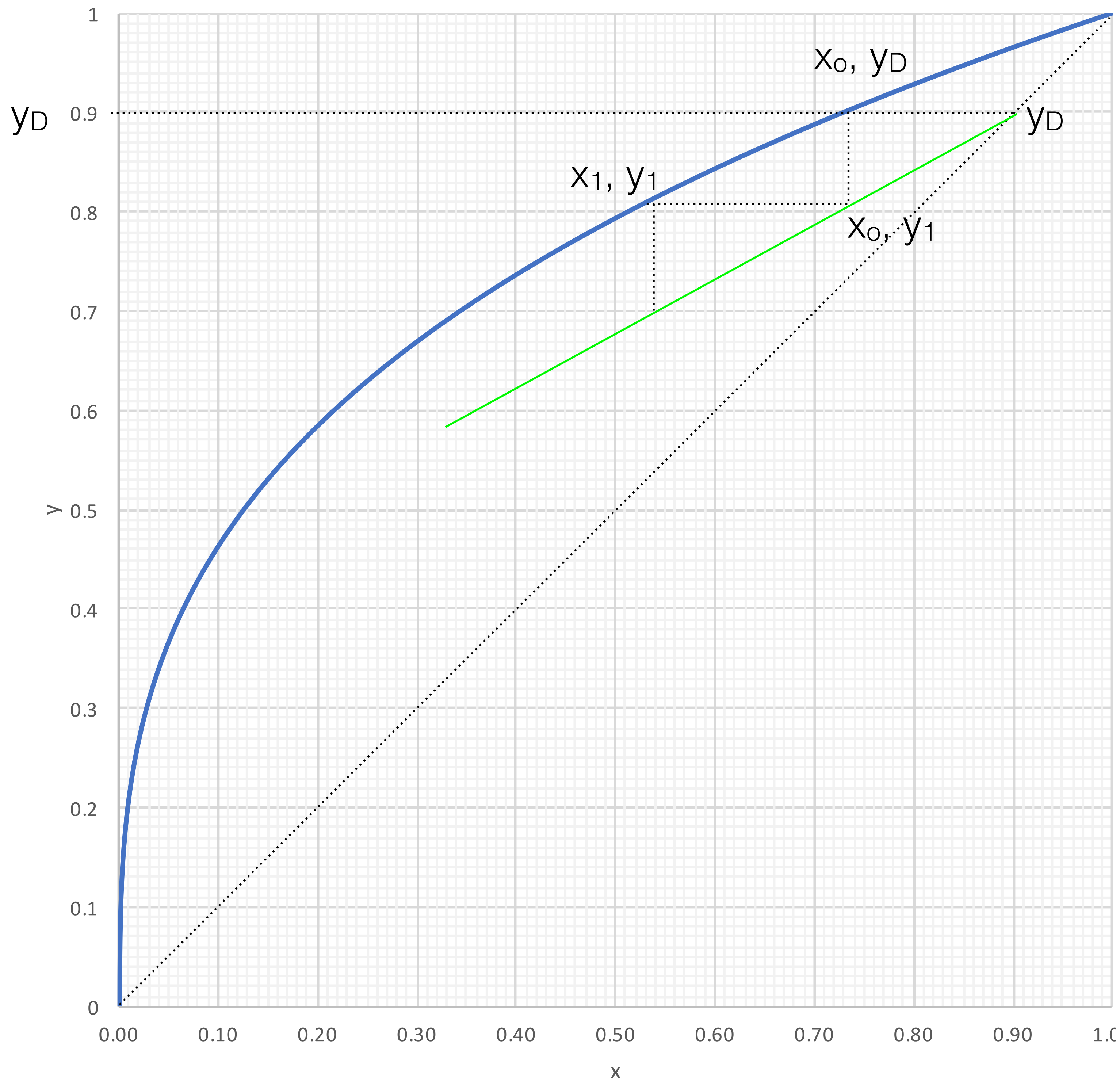


Partial condenser
D
Distillate vapor, yD
V
L
F, z
Stage N
B, xB
Total reboiler

The case of partial condenser



Partial condenser



$$Vy = Lx + Dy_D$$

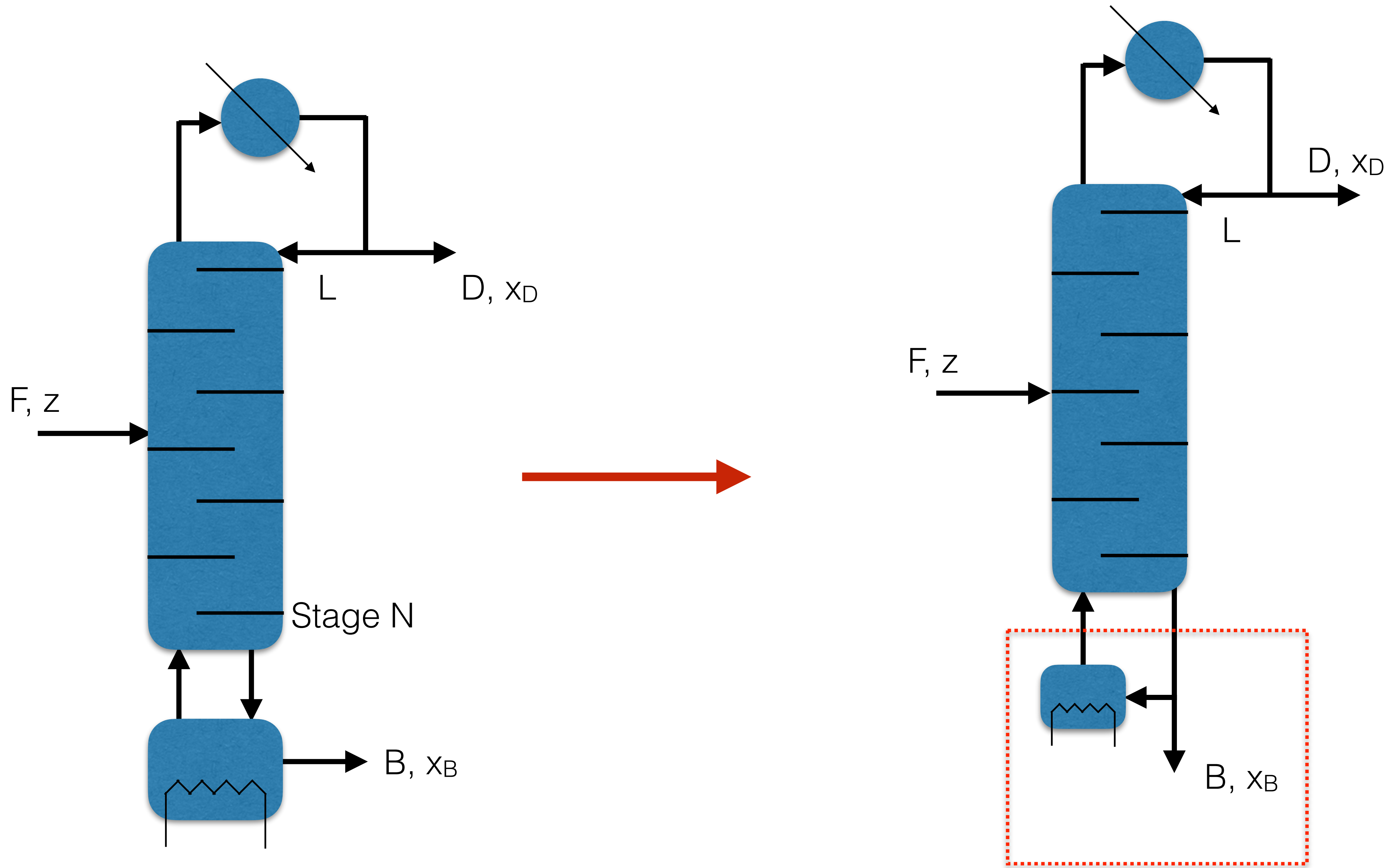
$$y = \frac{L}{V}x + \frac{D}{V}y_D$$

$$y = \frac{R}{R+1}x + \frac{1}{R+1}y_D$$

$$y = y_D$$

$$\Rightarrow x = y_D$$

The case of total reboiler



Partial reboiler

Total reboiler

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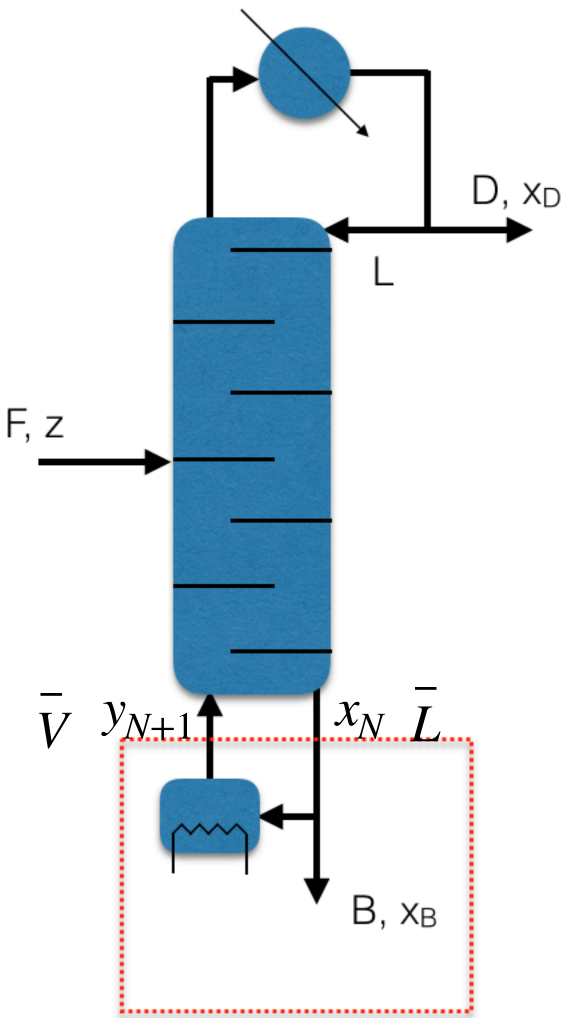
The case of total reboiler

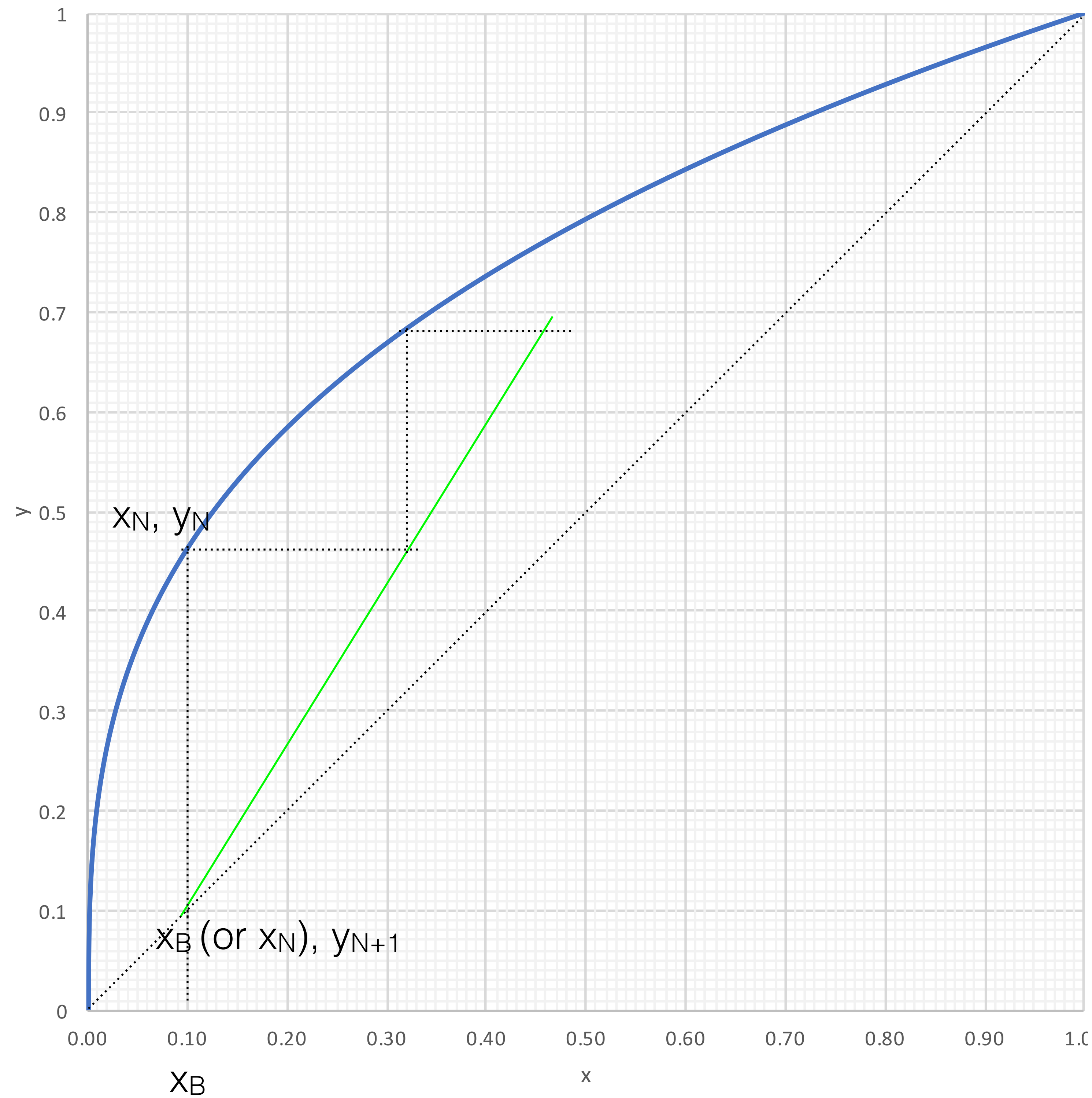


$$y_{N+1} = x_N = x_B$$

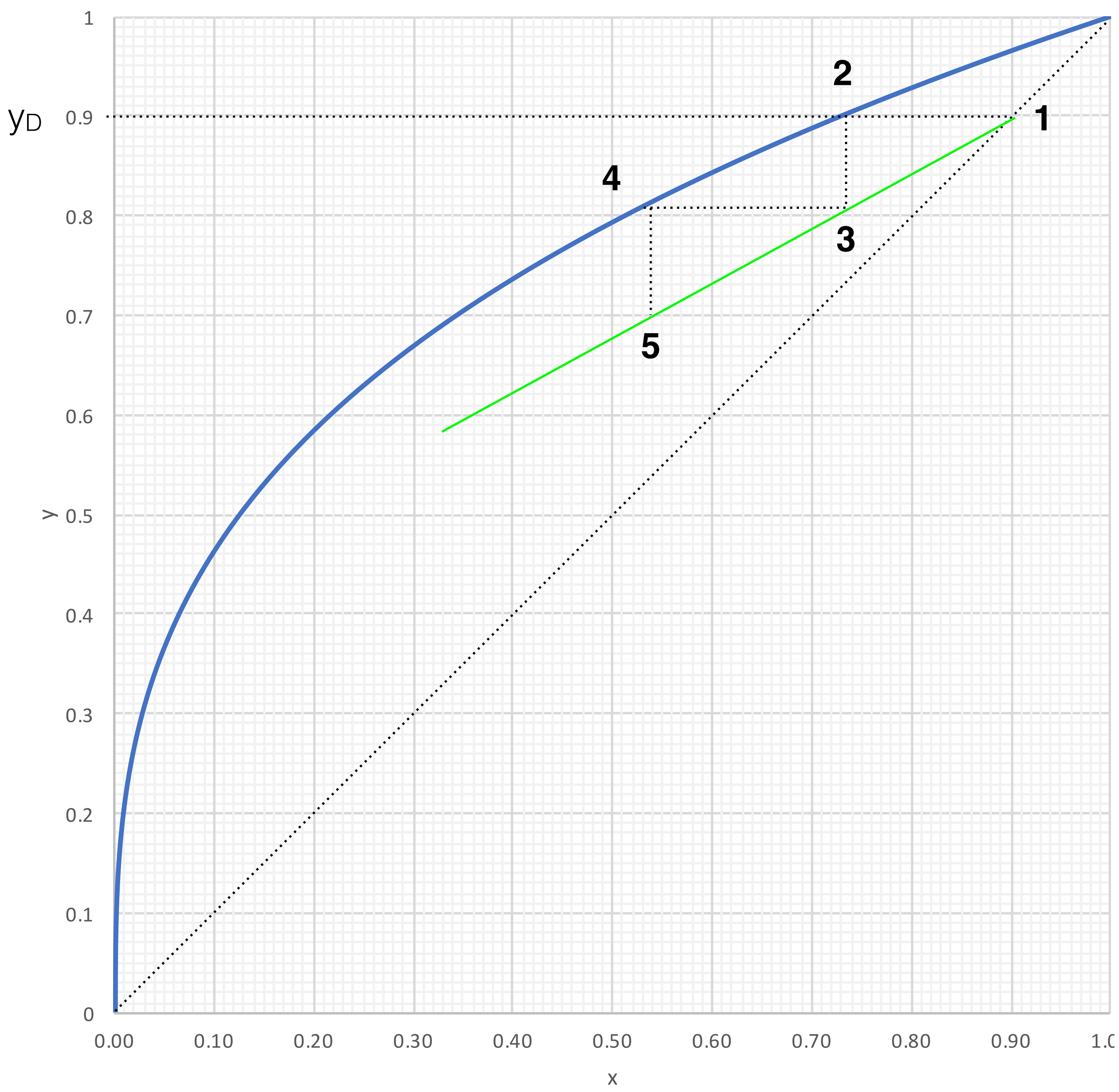
$$\bar{L} = \bar{V} + B$$

$$\bar{L}x = \bar{V}y + Bx_B$$

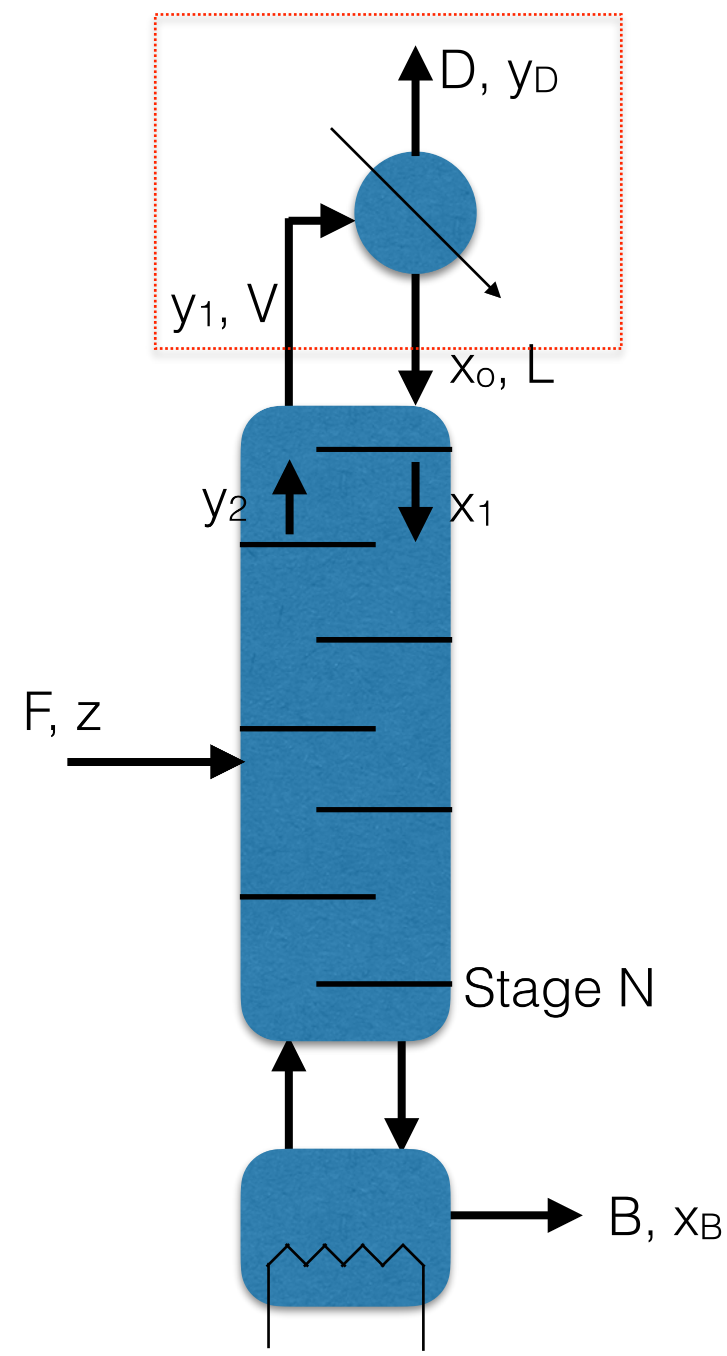
$$y = \frac{V_B + 1}{V_B}x - \frac{1}{V_B}x_B$$



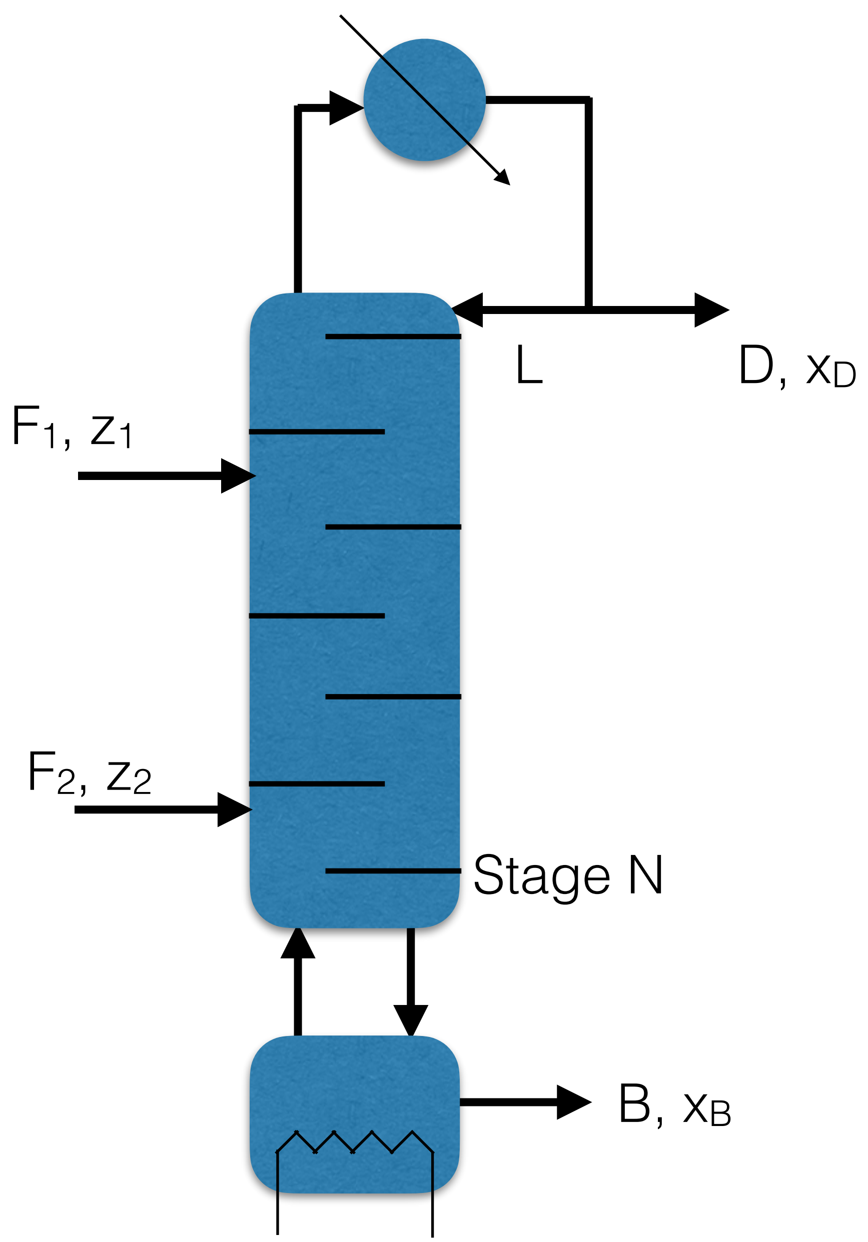
Which points on the plot represent (x_1, y_1)

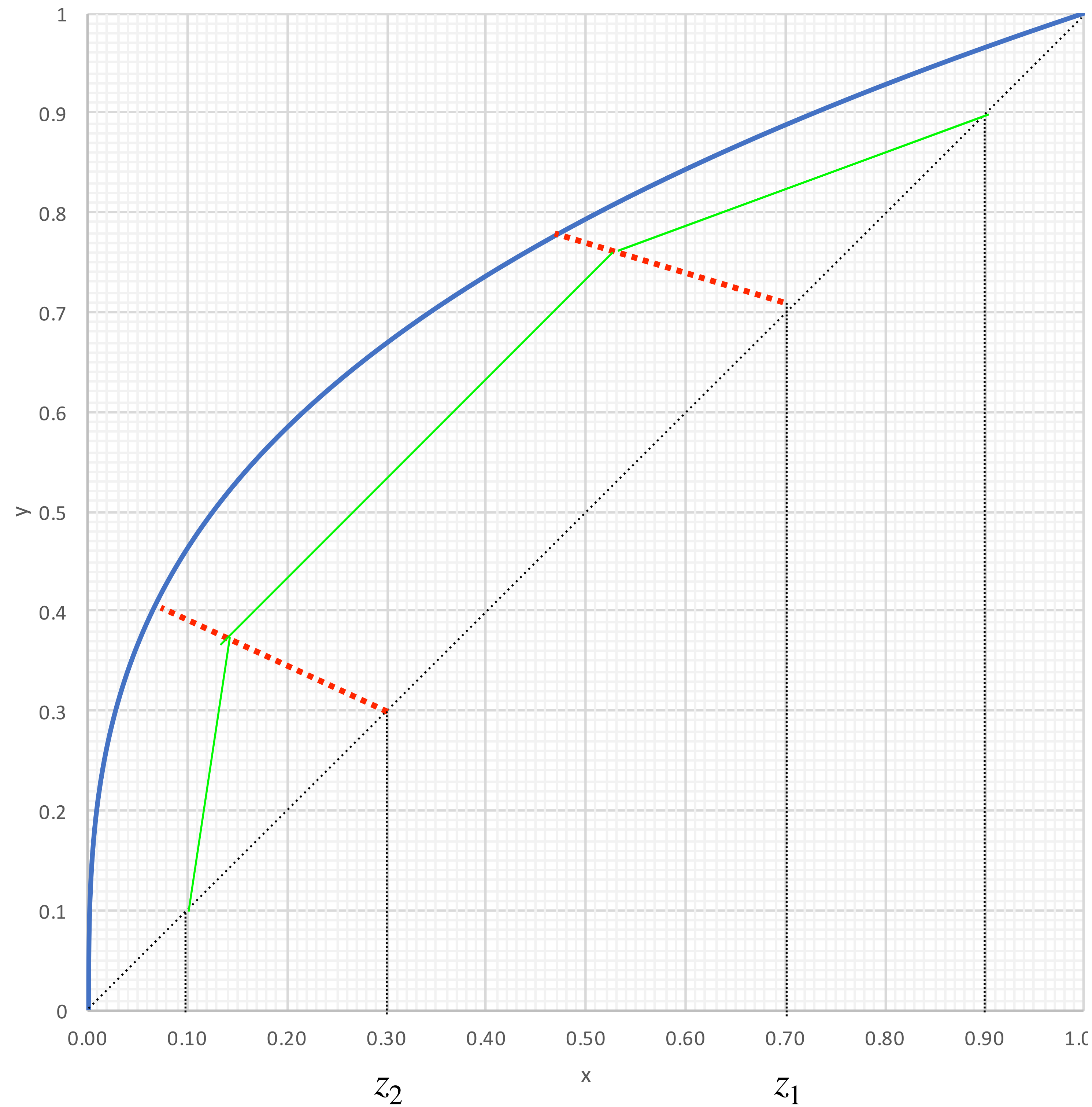


Partial condenser

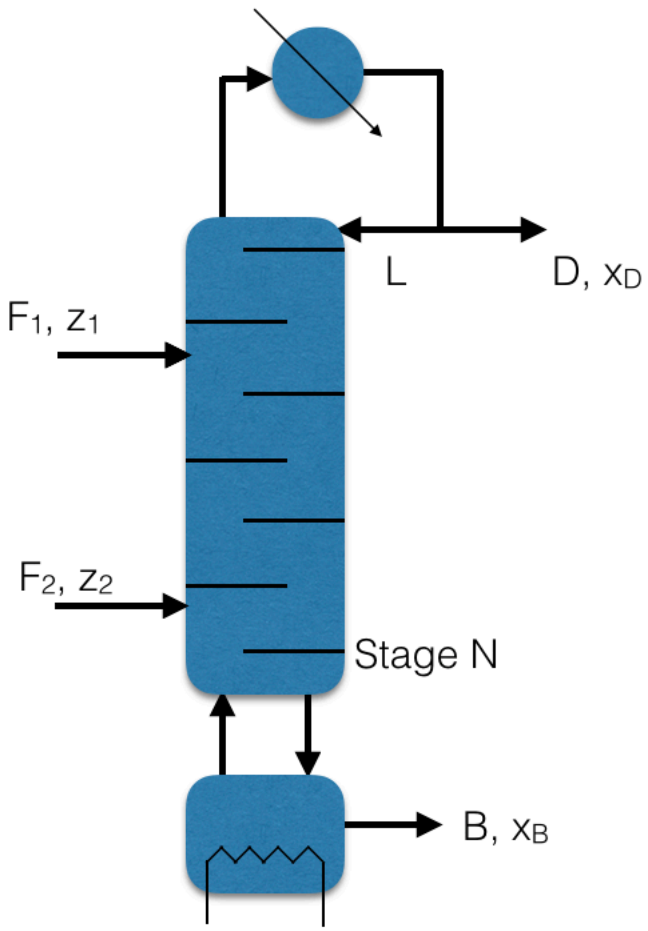


The case of multiple feeds





The case of multiple feeds and outlets



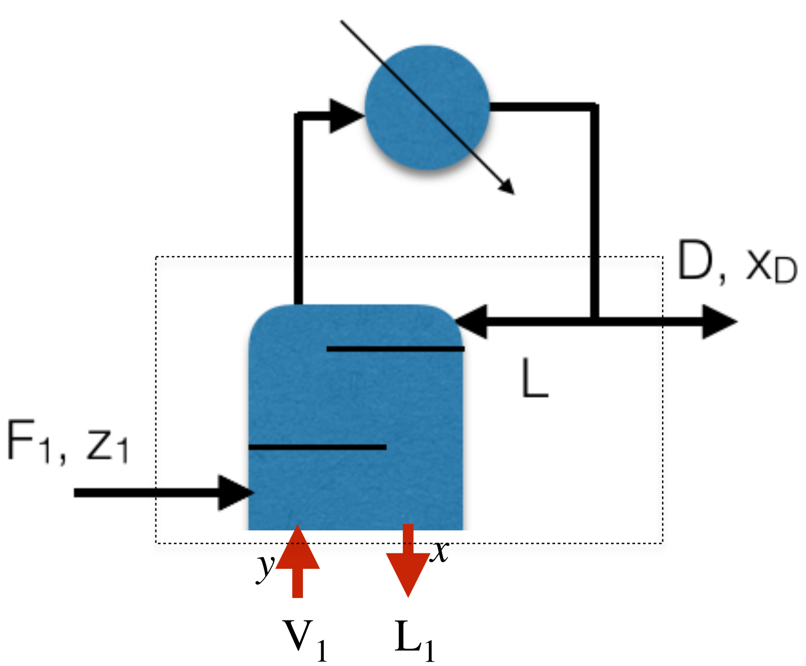
$$V_1 + F_1 = L_1 + D \qquad q_1 = \frac{L_1 - L}{F_1}$$

$$V_1 y + F_1 z_1 \; = \; L_1 x + D x_D$$

$$\Rightarrow V_1 + F_1 = L + q_1 F_1 + D$$

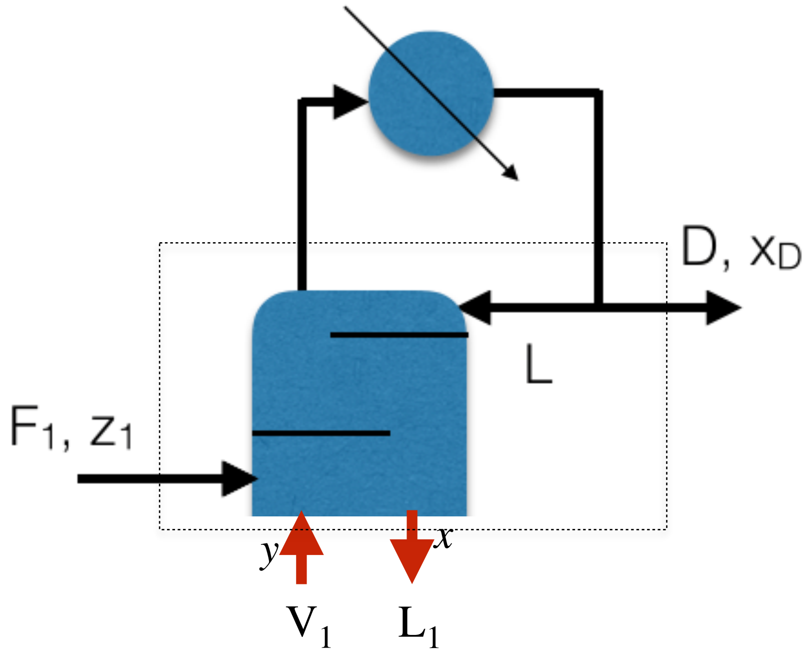
$$\Rightarrow V_1 y + F_1 z_1 \; = \; (L + q_1 F_1) x + D x_D$$

$$\Rightarrow y \; = \; \frac{L + q_1 F_1}{V_1} x + \frac{(D x_D - F_1 z_1)}{V_1}$$

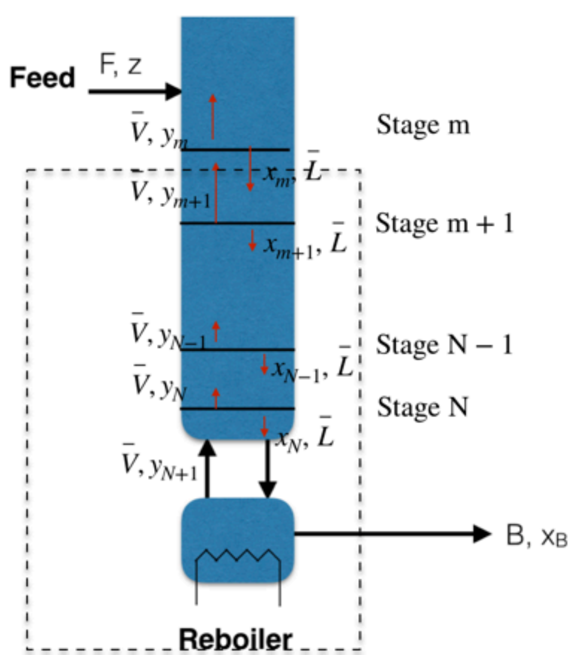


$$\Rightarrow y \; = \; \frac{(L + q_1 F_1)}{(L + q_1 F_1) + D - F_1} x + \frac{(D x_D - F_1 z_1)}{(L + q_1 F_1) + D - F_1}$$

Lets take a fresh look at operating line for stripping section



$$y = \frac{(L + q_1F_1)}{(L + q_1F_1) + D - F_1}x + \frac{(Dx_D - F_1z_1)}{(L + q_1F_1) + D - F_1}$$



$$y = \frac{V_B + 1}{V_B}x - \frac{1}{V_B}x_B$$

Above two equations are similar if we have only 1 feed

Can you prove it ?

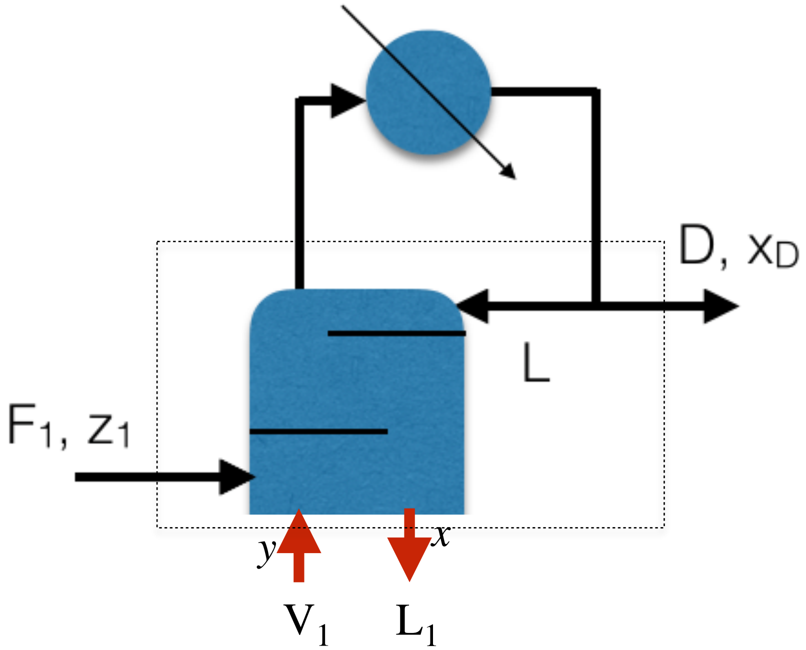
for single feed, $V_1 = \bar{V}$

$q_1 = q; \quad F_1 = F$

Lets take a fresh look at operating line for stripping section

for single feed, $V_1 = \bar{V}, L_1 = \bar{L}$

$q_1 = q; \quad F_1 = F$

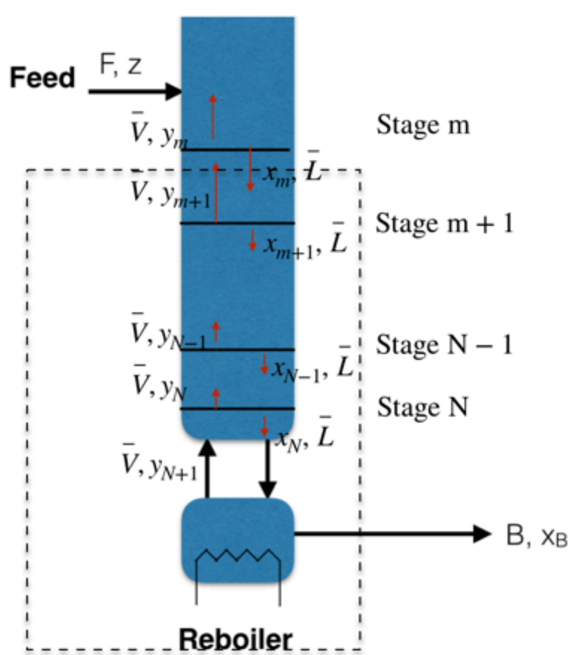


$$q = \frac{\bar{L} - L}{F} = 1 - \frac{V - \bar{V}}{F}$$

$$\Rightarrow \bar{V} = L + D + qF - F \qquad \Rightarrow V_B = \frac{\bar{V}}{B} = \frac{L + D + qF - F}{B}$$

Operating line

$$y = \frac{V_B + 1}{V_B} x - \frac{1}{V_B} x_B$$

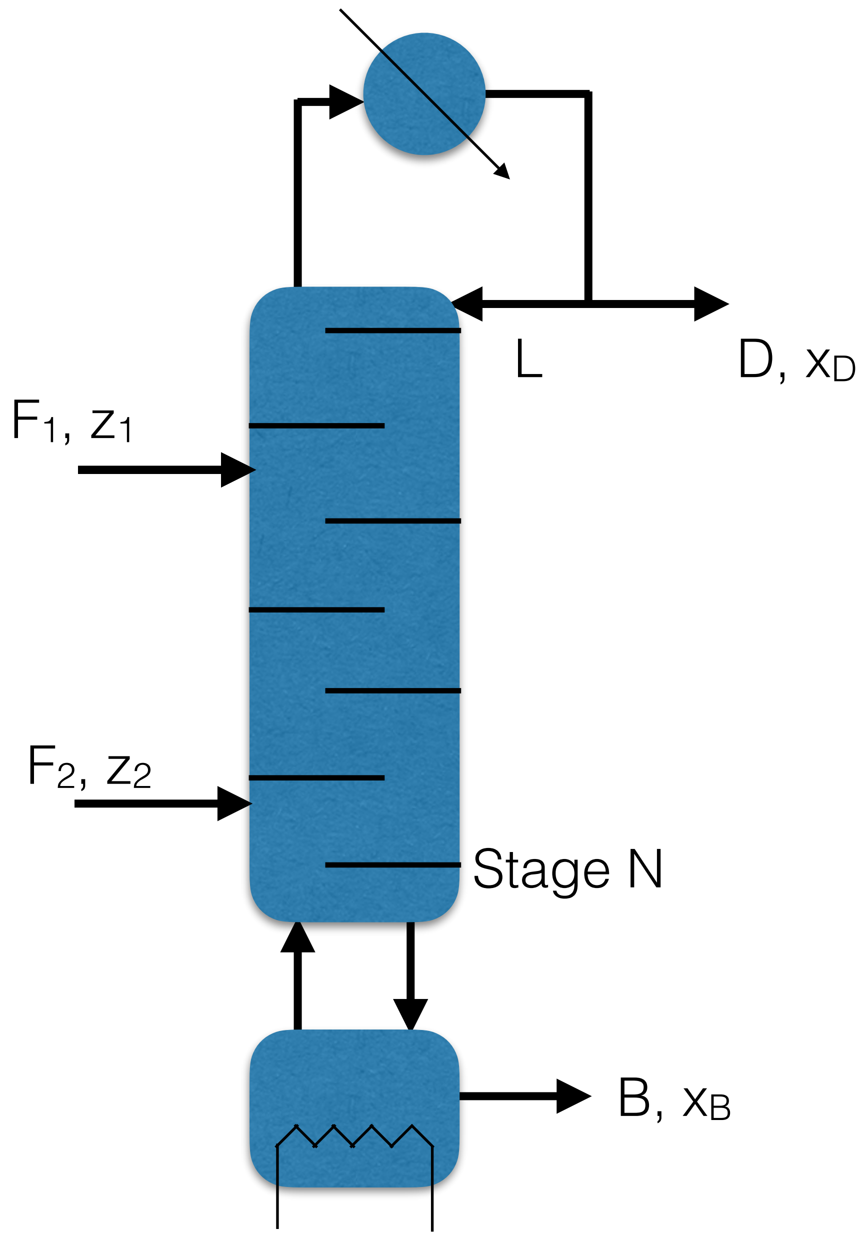


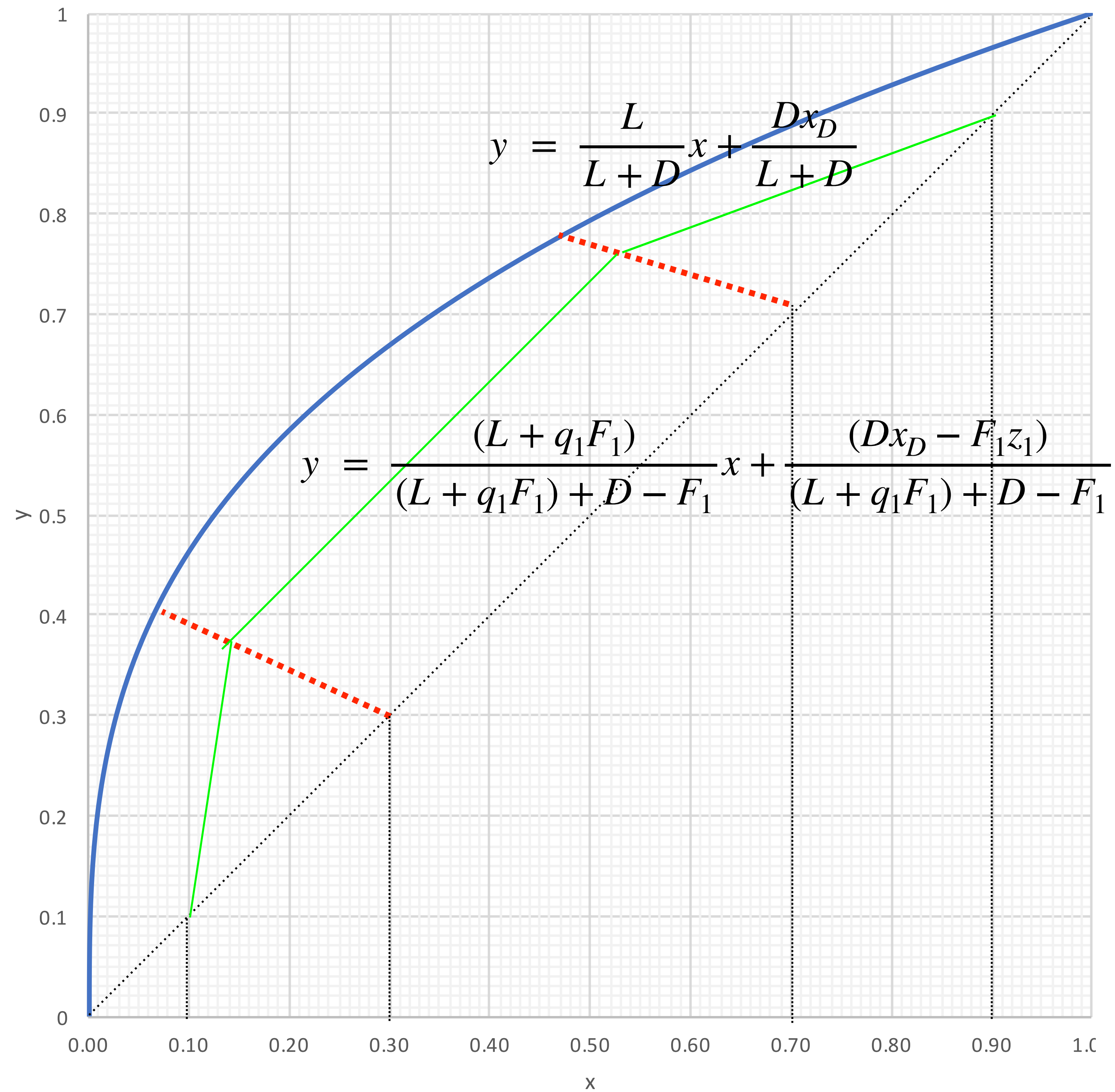
$$y = \frac{L + \cancel{D} + qF - \cancel{F} + \cancel{B}}{L + D + qF - F} x - \frac{Bx_B}{L + D + qF - F}$$

mass balance $Fz = Dx_D + Bx_B$

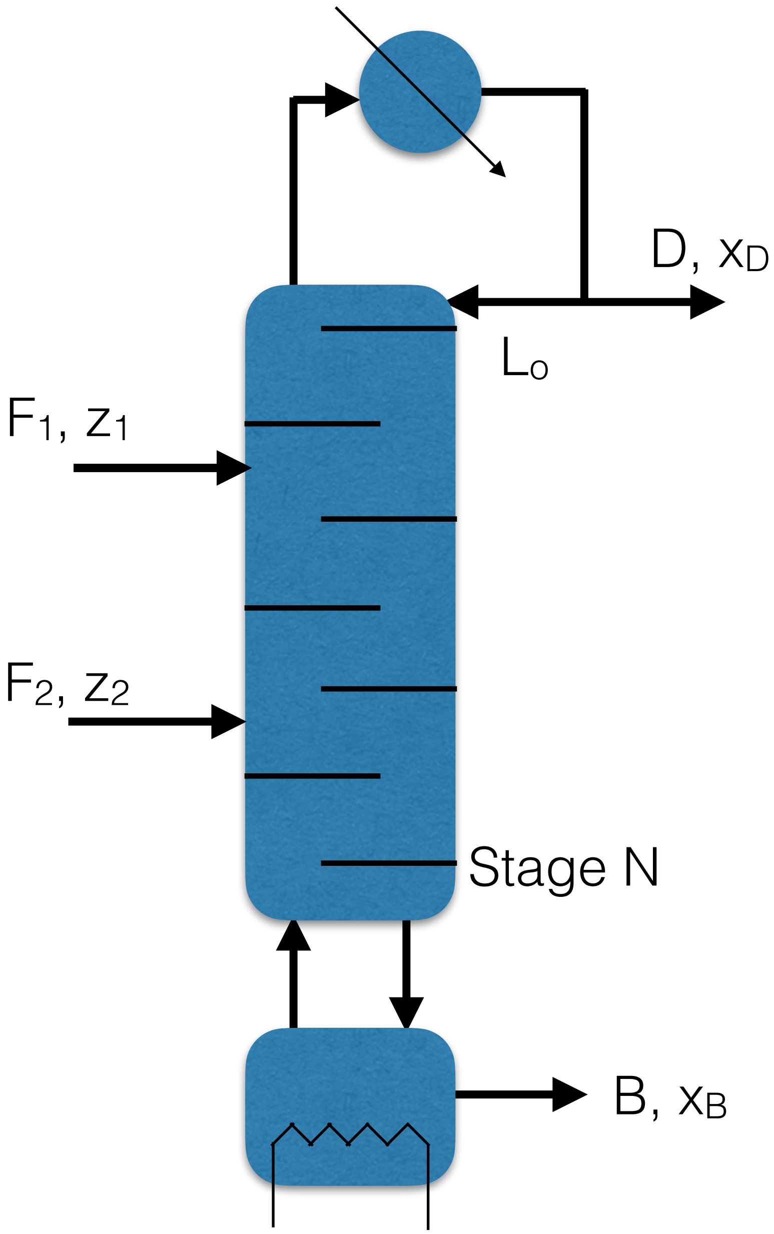
$$y = \frac{(L + qF)}{(L + qF) + D - F} x + \frac{(Dx_D - Fz)}{(L + qF) + D - F}$$

The case of multiple feeds



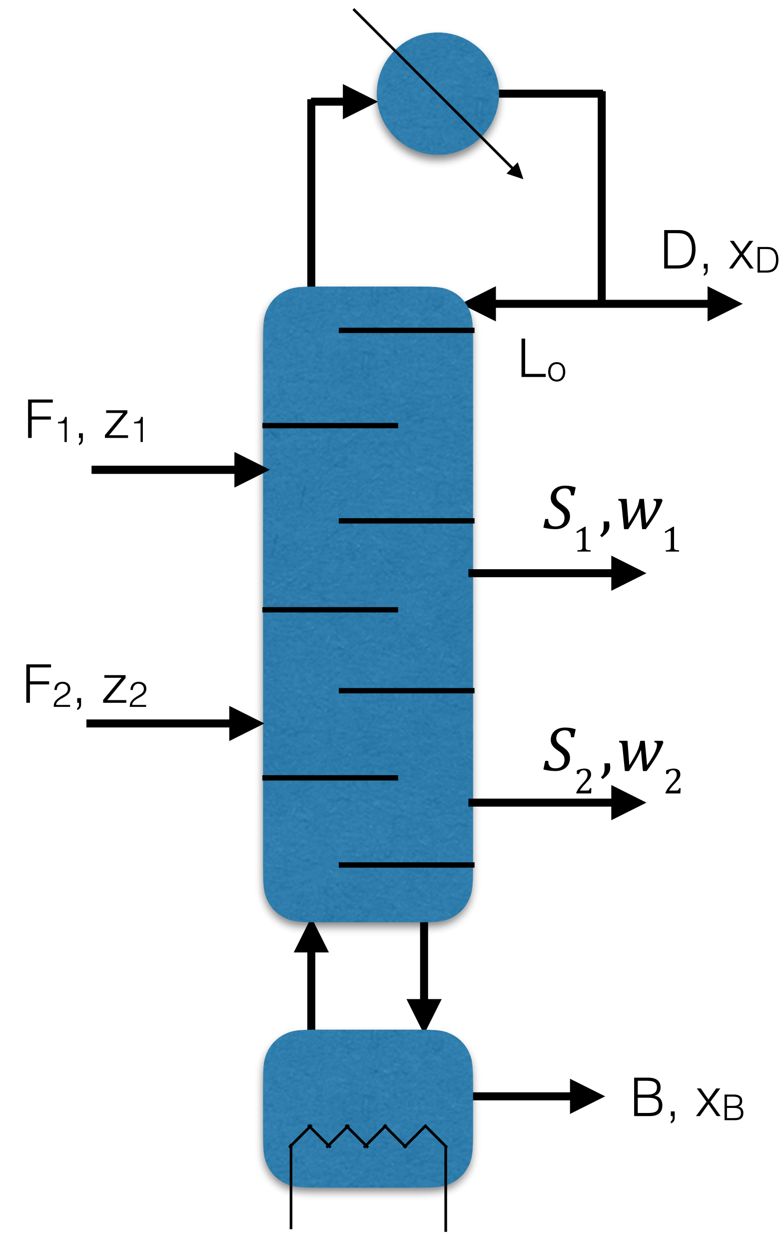


The case of multiple feeds and outlets



below g feed

$$y_g = \frac{RD + \sum_{i=1}^{g} q_i F_i}{RD + \sum_{i=1}^{g} q_i F_i + D - \sum_{i=1}^{g} F_i} x_g + \frac{D x_D - \sum_{i=1}^{g} z_i F_i}{RD + \sum_{i=1}^{g} q_i F_i + D - \sum_{i=1}^{g} F_i}$$



below g feed and k outlets

$$y_{g+k} = \frac{RD + \sum_{i=1}^{g} q_i^F F_i - \sum_{i=1}^{k} q_i^s S_i}{RD + \sum_{i=1}^{g} q_i^F F_i - \sum_{i=1}^{k} q_i^s S_i + D - \left(\sum_{i=1}^{g} F_i - \sum_{i=1}^{k} S_i\right)} x_{g+k} + \frac{D x_D - \left(\sum_{i=1}^{g} z_i F_i - \sum_{i=1}^{k} w_i S_i\right)}{RD + \sum_{i=1}^{g} q_i^F F_i - \sum_{i=1}^{k} q_i^s S_i + D - \left(\sum_{i=1}^{g} F_i - \sum_{i=1}^{k} S_i\right)}$$

Overall tray efficiency

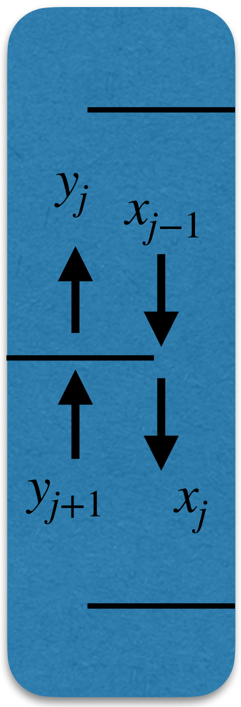
$$\eta = \frac{\text{number of theoretical trays (McCabe} - \text{Thiele method)}}{\text{number of actual trays}}$$

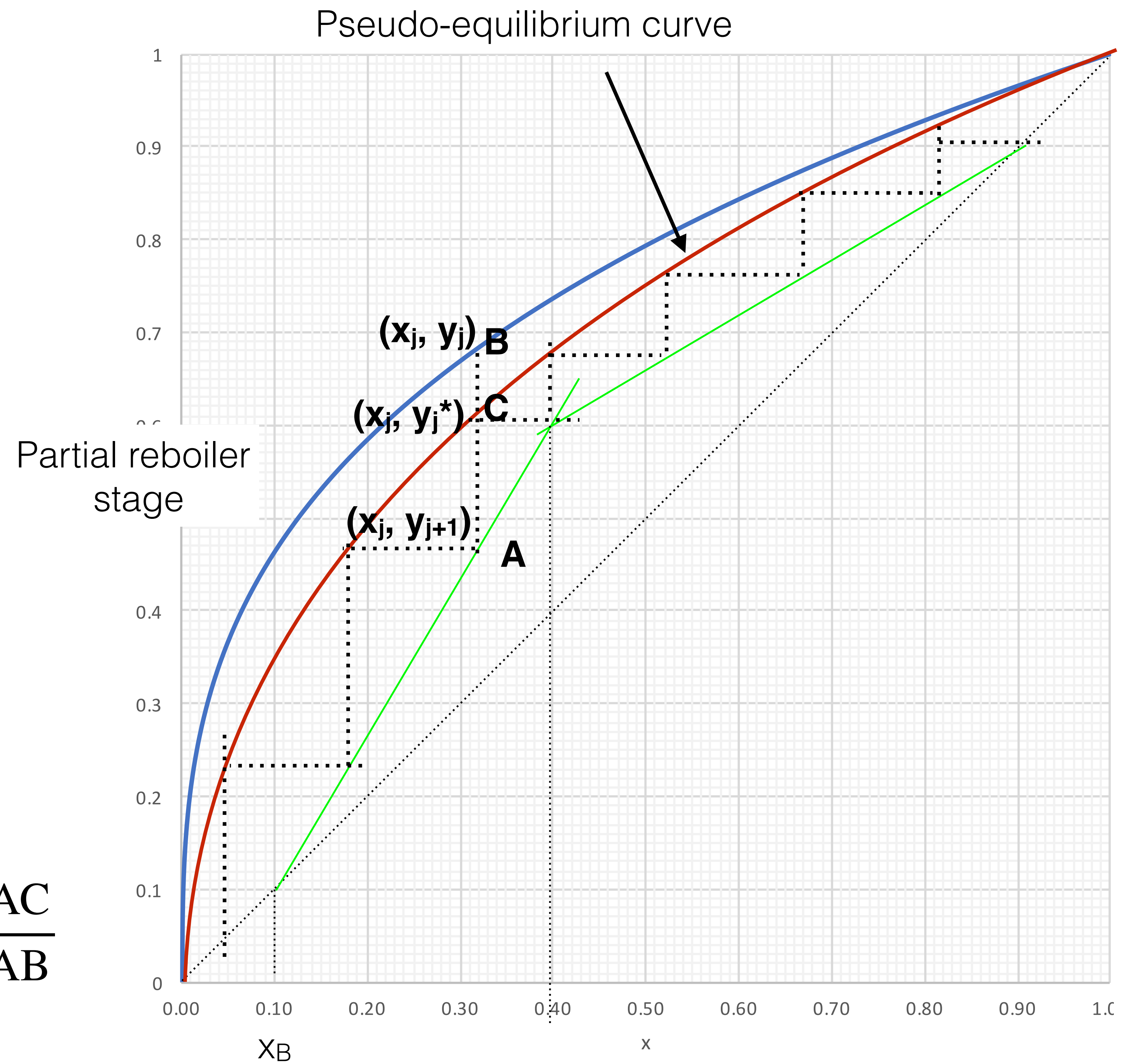
Every tray is assumed to have the same efficiency

The efficiency depends upon

1) geometry and design of the contacting trays.
2) Flow rates and flow paths of vapor and liquid streams.
3) Composition and properties of vapor and liquid streams.

Murphee Stage efficiency for vapor



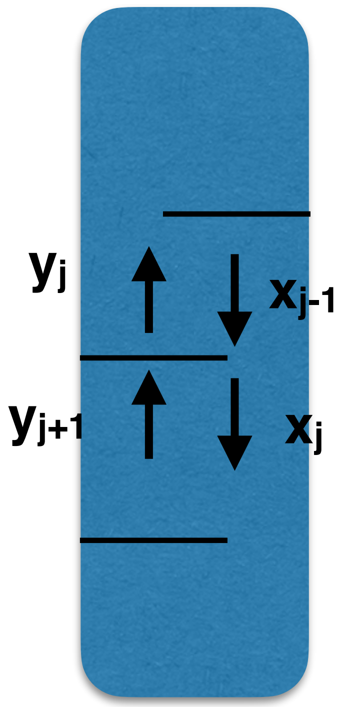


y_j is not in equilibrium with x_j

y_j^* is in equilibrium with x_j

$$\eta_{\text{murphee-vapor}} = \frac{y_j^* - y_{j+1}}{y_j - y_{j+1}} = \frac{AC}{AB}$$

Murphee Stage efficiency for liquid



yj is not in equilibrium with xj
yj is in equilibrium with xj*

$$\eta = \frac{x_j^* - x_{j+1}}{x_j - x_{j+1}} = \frac{AC}{AB}$$